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Prospective control in an enhanced manpower planning model

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ABSTRACT

The present paper deals with the exercise of recruitment control to a time dependent, hierarchical system which incorporates training classes as well as two streams of recruitment; one coming from the outside environment and another from an auxiliary external system. The motivation for this model lies in the need to take into account not only the tendency of the employees to attend seminar courses so as to improve their career prospects, but also the organizations' intention to avoid situations associated with the unavailability of skilled individuals for hiring. For the suggested model, we define its attainable structures and the sets containing these structures, i.e. attainable sets. We examine the geometrical properties of these sets and it is proved that they form convex polytopes. Moreover, by adopting the equivalent approach of describing convex polyhedral sets via inequalities, we specify the attainable sets of the model for every time point *t*. An illustrative example follows.

1. Introduction

Nowadays, it is generally accepted that the use of mathematical models for manpower planning is well established. In manpower planning we are concerned with the description and prediction of the behavior of large numbers of people. Such data is well suited to a mathematical modeling approach since, although individual behavior is unpredictable, when aggregated, the data is seen to follow probabilistic patterns [1], especially the ones represented by Markov chains. Ever since Seal [2] first applied the principles of Markov chains to describe the behavior of a manpower structure, there has been enormous development and utilization of these models. In all cases, the initial target was the adequate description of system's population dynamics, such as hiring, attrition, promotion and retirement, as well as of the influences of the environment in which the system operates. In turn, these models assist in the successful planning and control of manpower interrelated activities, thus improving the system's capability to meet its strategic objectives over time. An extensive list of references as well as an extended discussion on related material can be found in Bartholomew [3], Bartholomew et al. [4], Ugwuowo and McClean [5], Wang [6] and De Feyter [7].

A short introduction of the basic notation and formulas of a Markov manpower planning system model is provided here, since the material is extensively presented in numerous articles and books mentioned in the references. Traditionally, consider a hierarchical, time depended population structure which is stratified into a finite number of classes (states, grades), say *k*, on the basis of whatever homogeneous attributes are relevant for the problem at hand. The classes are

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assumed to be exclusive and exhaustive, while we number them serially with the base level class to be class 1 and the uppermost class to be class k. The time frame of reference varies over discrete values of $t = 0, 1, \ldots$ with $t = 0$ representing the initial point of time. At any time t, the row vector of the class expected levels $\mathbf{N}(t) = [N_1(t)\ N_2(t)\ \ldots\ N_k(t)]$ provides a snapshot of the system's expected structure. Assume that the individual transitions between the states take place according to a time dependent Markov chain and let $\{\mathbf{P}(t)\}_{t=0}^{\infty}$ be the sequence of transition probability matrices between the states at t-th time interval $[t, t+1)$. For the t-th time interval, let also $\{\mathbf{p}_{k+1}(t)\}_{t=0}^{\infty}$ be the sequence of the vectors of loss probabilities and $\{\mathbf{p}_0(t)\}_{t=0}^{\infty}$ the sequence of input probabilities. At each time point t, the expected total size of the system $T(t) = \mathbf{N}(t)\mathbf{1}'$ where $\mathbf{1} = [1 \ldots 1]$ is determined in advance; a full discussion of Markov models with given size (expanding or not) is sited on Bartholomew [3, p. 72] and Bartholomew et al. [4, p. 103]. The transition matrix, the wastage rates and the recruitment vector, define the embedded non-homogeneous Markov chain, represented by a sequence of stochastic matrices, namely $\mathbf{Q}(t) = \mathbf{P}(t) + \mathbf{p}'_{k+1}(t)\mathbf{p}_0(t)$. This framework is known as the k-state Non-Homogeneous Markov system (NHMS, Vassiliou [8,9]). The difference equation $\mathbf{N}(t) = \mathbf{N}(t-1)\mathbf{Q}(t-1) + \mathbf{p}_0(t-1)[T(t) - T(t-1)]$, gives the expected structure of the system at time t, as a function of its structure at time $t-1$. This model and other like this were extensively utilized for prediction, control or optimization, in view of certain recruitment or promotion policies. In this respect, various studies dealt with the proper selection of the control variables, so as to accomplish a prescribed personnel size for a given set of grades. In addition to the above, researchers examined the problems of asymptotic behavior under particular assumptions, of attainability and maintainability, of constrained optimization, of the issues appearing in stochastic environments etc; the review papers [9,5,6] give an early but decent account.

Lately, new ground has been broken in the area of mathematical models for manpower planning. An interesting idea was proposed by De Feyter [10], who divided the studied system's population in several more homogeneous subgroups. This gives the opportunity of a deeper and probably more sophisticated research. Nilakantan and Raghavendra [11] suggested a "proportionality" policy to a Markov manpower system, i.e. the recruitment to every level of the hierarchy (except the bottom most level) to be in strict proportion to the promotions into that level. Another idea was presented by Georgiou and Tsantas [12] who enriched the NHMS framework with an external state next to the original/active internal classes. The meaning of this new class, called trainee class, was to give mathematical meaning to a proportion of newcomers who receive training for a transient period before they become active members of the system. In their proposed *Augmented Mobility Model* (AMM), they went on studying cost optimization policies using a goal programming approach. The work of this paper has its roots in the latter AMM model and is motivated by the fact that since the late nineties, the European Union has been promoting regional and local governmental assistance programs. These programs have been exploited by firms mainly to improve the skills and knowledge of their existing personnel. In addition, many organizations have faced the problem, especially at periods of excessive demand, of not finding available specifically skilled and/or experienced persons for hiring. In this respect we employ a new modeling approach that incorporates these factors into a Markov chain manpower planning model. This approach, named the *Generalized Augmented Mobility Model* (GAMM), constitutes an enhancement of the *Augmented Mobility Model* (AMM, [12]), in the following sense:

- Firstly, GAMM takes into account the fact that in many companies the employees may attend a training course or a seminar, before being promoted. In this context, we insert new classes in the *active classes* of the AMM, called *training classes*, which represent the company's policy to train its already existing personnel so as to acquire knowledge and skills needed for a smooth promotion. These training classes along with the grades of the system form its *internal classes*.
- Secondly, there is an additional personnel's *preparation class* which is treated as the representative class of an *external* Markov system. It is assumed that this system works in cooperation with the system consisting of the internal classes, and involves individuals that although they possess the required degree or degrees for a post, they lack experience and/or specialized skills for that post. Therefore, these people participate voluntarily in preparation class, so as to obtain particular knowledge, instructions and skills for the system's posts from expert members and consultants of the firm. After completing their preparation, staying for at least one time unit in preparation class, they are qualified enough for being potential members of the system and anticipate to be hired as new recruits.
- Finally, GAMM permits the members of the preparation class to abort their preparation. This is the real case when, for instance, an individual of the preparation class finds a job in another organization while he/she was waiting to be hired by the system.

In the present uncertain and eternal changing economic environment (think the current recession) the organizations have an extra need to foresee which may be the population structures the system may end up under various policies (which are unpredictable from now) after a certain time period. This fact is of vital importance in practice since the Manpower Planner is obliged to take the necessary measures during this period in order to avoid personnel structures which will not allow the company to get all the work done (satisfy the current demand), as there may be inappropriate number of employees in some posts. Therefore, the analysis of the way manpower policies affect the personnel structures of the organizations constitutes a fundamental goal in human resource management. As the literature in manpower systems shows [3, p. 151], the most attractive policy in controlling workforce flows is recruitment control. The control of an orga-