



Bifurcation analysis in an SIR epidemic model with birth pulse and pulse vaccination

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ABSTRACT

The dynamical behavior of an SIR epidemic model with birth pulse and pulse vaccination is discussed by means of both theoretical and numerical ways. This paper investigates the existence and stability of the infection-free periodic solution and the epidemic periodic solution. By using the impulsive effects, a Poincaré map is obtained. The Poincaré map, center manifold theorem, and bifurcation theorem are used to discuss flip bifurcation and bifurcation of the epidemic periodic solution. Moreover, the numerical results show that the epidemic periodic solution (period-one) bifurcates from the infection-free periodic solution through a supercritical bifurcation, the period-two solution bifurcates from the epidemic periodic solution through flip bifurcation, and the chaotic solution generated via a cascade of period-doubling bifurcations, which are in good agreement with the theoretical analysis.

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1. Introduction

The SIR epidemic model is an important model and has been studied by many authors. The ordinary differential equations (ODEs) were used to build SIR epidemic models in most cases, see [1] for example. However, impulsive differential equations (IDEs) [2,3] are suitable for the mathematical simulation of evolutionary processes in which the parameters undergo relatively long periods of smooth variation followed by a short-term rapid change in their values. Recent years have seen many results about the SIR epidemic model, which built by IDEs.

The pulse vaccination strategy (PVS), whose theoretical study was firstly proposed by Agur and coworkers in [4], consists of periodical repetitions of impulsive vaccinations in a population in contrast to the traditional constant vaccination. At each vaccination time, the vaccine dose is applied in a very short span of time and a constant fraction of susceptible population is vaccinated. The PVS was considered in many literatures, for example, D'Onofrio [5] obtained the global asymptotic stability of the eradication solution, Gakkhar and Negi [6] discussed the local and global stability of the infection-free periodic solution, Meng and Chen [7] obtained the stability of the infection-free periodic solution and discussed the superiority of the PVS. One example of successful application of the PVS is the UK vaccination campaign against measles in 1994 [8].

Traditional mathematical models of population dynamics have invariably assumed that the species reproduce throughout the year. However, many species give birth only during a single period of the year and Caughley [9] termed this growth pattern as birth pulse. Thus the continuous reproduction of population is then removed from the traditional models and replaced with a birth pulse. Roberts and Kao [10] proposed a model for the dynamics of a fatal infectious disease and discussed the existence and stability of periodic solutions. Tang and Chen [11] obtained an explicit periodic solution and the

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threshold conditions for its stability by using the stroboscopic map in a single-species model. Liu and Chen [12] obtained the existence of positive periodic solution in a system with birth pulse.

In the above papers, the study of *SIR* model mainly concerns the global asymptotic stability of solutions, however, little is known about the bifurcation theory of *SIR* model. Lakmeche et al. [13] transformed the problem of periodic solution into a fixed-point problem and obtained the conditions for the stability of trivial solution and the existence of positive period-one solution. This method limits to two state variables and many authors, see [6,14] for example, discussed the bifurcation of nontrivial periodic solution of *SIR* model by using the results obtained in [13]. By using the explicit solution, a stroboscopic map is obtained and used to discuss the bifurcation of periodic solution in [11]. Georgescu et al. [15] discussed the bifurcation of nontrivial periodic solutions via a projection method. However, there exist three state variables in an *SIR* model and it is not easy to obtain the explicit solution. Thus the stroboscopic map is not easy to be obtained and the bifurcation of periodic solution is difficult to be discussed in detail. Some authors investigated the complex dynamics of epidemic models, such as period-doubling bifurcation, chaos and crisis, etc., by means of numerical way, see [6] for example. Therefore, theoretical analyzing the bifurcation theory of *SIR* model is a challenging task. On the other hand, birth pulse and pulse vaccination strategy are seldom considered in the same model in the past. In this paper, not only birth pulse but also pulse vaccination is used to build an *SIR* epidemic model.

To study this epidemic model, we construct the Poincaré map and present analytical results about the complex dynamical behavior. The system is systematically investigated, including periodic attractors, derivation of two complete mathematical characterizations for flip bifurcation and bifurcation of nontrivial periodic solution, and numerical justification for theoretical results. The rest of the paper is organized as follows. In the next section, an *SIR* epidemic model with birth pulses and birth vaccination is introduced. The existence and stability of the infection-free periodic solution is discussed in Section 3. Bifurcation of nontrivial periodic solution is discussed in Section 4. In Section 5, the Poincaré map is constructed and the flip bifurcation is investigated by using center manifold theorem and bifurcation theorem. In Section 6, the numerical results including the periodic solutions and the bifurcation diagrams, are given by one example and the conclusion is presented in Section 7 finally.

2. Model description

Let $S(t)$, $I(t)$, and $R(t)$ denote the numbers of susceptible, infective, and removed individuals at time t , respectively. The following *SIR* epidemic model is well studied.

$$\begin{cases} \dot{S} = \mu - \sigma S - \beta SI + \delta R, \\ \dot{I} = \beta SI - (\gamma + \sigma)I, \\ \dot{R} = \gamma I - (\delta + \sigma)R, \end{cases} \quad (1)$$

where the constant μ is the recruitment rate, σ is the natural death rate, δ is the rate at which infective individuals loose immunity and return to the susceptible class, and γ is the natural recovery rate of the infective population. Susceptible become infectious at a rate βI , where β is the contact rate. The *SIR* epidemic model with horizontal and vertical transmission was studied by many authors. For simplicity, vertical transmission is not considered in our case.

In system (1), μ represents a constant birth rate, which means that dynamics increase in population due to birth are assumed to be time-independent. But many species give birth only during a single period of the year and this growth pattern is called birth pulse. In most cases, the birth pulse is assumed to be the linear birth pulse $\Delta N = pN$ [12], where $N = S + I + R$. Roberts and Kao considered the birth pulse $\Delta N = B(N)N$, $B(N) = b - cN^q$ in [10] while Tang and Chen considered the birth pulse $B(N) = b \exp(N) - 1$ and $B(N) = \frac{p}{q+N^q} - 1$ in [11]. In this paper, the birth pulse is taken as $\Delta N = (b - cN)N$, where $c = r(b - d)$, b is the maximum birth rate, d is the maximum death rate, r is a parameter reflecting the relative importance of density-dependent population regulation through births and deaths. The newborn population is assumed to be susceptible of disease, that is, $\Delta S = (b - cN)N$ and $\Delta I = 0$.

At each vaccination time, a constant fraction p of susceptible population is vaccinated under the PVS, that is $\Delta S = -pS$, $\Delta R = pS$, where $0 < p < 1$. For simplicity, the birth pulse and pulse vaccination are assumed to be occurred at the same time $t = nT$, where $n \in \mathbb{N}_+$ and T is the time between two consecutive pulse vaccinations. Then we obtain the following epidemic model with birth pulse and pulse vaccination:

$$\begin{cases} \dot{S} = -\sigma S - \beta SI + \delta R, \\ \dot{I} = \beta SI - (\gamma + \sigma)I, \\ \dot{R} = \gamma I - (\delta + \sigma)R, \end{cases} \quad t \neq nT, \\ \begin{cases} \Delta S = (b - cN)N - pS, \\ \Delta I = 0, \\ \Delta R = pS \end{cases} \quad t = nT, \end{cases} \quad (2)$$

where $N = S + I + R$, the meanings of parameters β , σ , δ , and γ are the same as in model (1), $\Delta S(t) = S(t^+) - S(t)$, $0 < p < 1$, $S(t^+) = \lim_{\tau \rightarrow 0^+} S(t + \tau)$, $\Delta I(t) = I(t^+) - I(t)$, $I(t^+) = \lim_{\tau \rightarrow 0^+} I(t + \tau)$, $\Delta R(t) = R(t^+) - R(t)$, $R(t^+) = \lim_{\tau \rightarrow 0^+} R(t + \tau)$. More details about impulsive system see [1].

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