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Traveling wave fronts in a delayed population model of Daphnia magna

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ABSTRACT

This paper deals with the traveling wave fronts of a delayed population model with nonlocal dispersal. By constructing proper upper and lower solutions, the existence of the traveling wave fronts is proved. In particular, we show such a traveling wave front is strictly monotone.

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1. Introduction

In population dynamics, one cornerstone is the famous Logistic equation as follows:

$$\frac{du(t)}{dt} = ru(t)[1 - u(t)],$$

in which $r > 0$ is a positive constant, and there are many mathematical results on such a model and its delayed version [14]. Based on the Logistic model and the experiment results of the density evolution of Daphnia magna, Smith [18] proposed the following population model

$$\frac{du(t)}{dt} = ru(t)\left[\frac{1 - u(t)}{1 + \beta u(t)}\right], \tag{1.1}$$

where $\beta \geqslant 0$ is a constant independent of r. Eq. (1.1) is also called a food-limited population model and we can refer to Pielou [17]. Gopalsamy et al. [8] introduced time delay (also see Kuang [13] for more details on the delayed models) into (1.1) and got the delayed model as follows:

$$\frac{du(t)}{dt} = ru(t)\left[\frac{1 - u(t-\tau)}{1 + \beta u(t-\tau)}\right], \tag{1.2}$$

in which $\tau \geqslant 0$ is accounted for time delay, and the oscillation of the positive solution is investigated if $\tau > 0$. For a delayed model similar to (1.2), Gopalsamy et al. [9] and So and Yu [19] also studied it from the viewpoint of the periodic solutions and the stability of the positive equilibrium, respectively.

Due to the importance of the spatial dispersal, such as the spatial invasion of the species [10,20], many investigators also considered the corresponding spatially inhomogeneous model of (1.1) and (1.2). For example, Gourley [10] considered the traveling wave solutions of the following diffusion equation with time delay

$$\frac{\partial u(x,t)}{\partial t} = d\frac{\partial^2 u(x,t)}{\partial x^2} + ru(x,t)\left[\frac{1 - u(x,t-\tau)}{1 + \beta u(x,t-\tau)}\right] \tag{1.3}$$

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for $x \in \mathbb{R},\ d > 0$, herein a traveling wave solution is a special solution $u(x,t) = \phi(x+ct)$ with $c > 0$ and $\phi \in C^2(\mathbb{R},\mathbb{R})$. More precisely, the existence of monotone traveling wave solution was proved if time delay τ is small enough, and the traveling wave solution was studied numerically if time delay is large. Furthermore, Gourley and Chaplain [11] and Wang and Li [20] investigated the traveling wave fronts of the nonlocal delay version of (1.3). These models in [10,11,20] are defined on continuous spatial environment, we also refer to Feng and Lu [5,6] for more results. For the corresponding discrete patch environment of (1.3), Huang et al. [12] considered the following lattice differential equation:

$$\frac{dv_n(t)}{dt} = d[v_{n+1}(t) - 2v_n(t) + v_{n-1}(t)] + rv_n(t)\left[\frac{1 - v_n(t-\tau)}{1+\beta v_n(t-\tau)}\right], \tag{1.4}$$

where $n \in \mathbb{Z}$. The existence of traveling wave solution of such an equation was proved by constructing proper upper and lower solutions.

In this paper, we shall investigate the traveling wave front (monotone traveling wave solution) of another spatial migration pattern of (1.2) as follows:

$$\frac{\partial u(x,t)}{\partial t} = \int_{\mathbb{R}} J(y)[u(x-y,t) - u(x,t)]dy + ru(x,t)\left[\frac{1-u(x,t-\tau)}{1+\beta u(x,t-\tau)}\right], \tag{1.5}$$

in which $J : \mathbb{R} \to \mathbb{R}$ is a probability function describing the migration of the individuals. Different from the diffusion process formulated by Laplacian operator in Eq. (1.3), such a diffusion pattern is useful in describing the long range effect in population dynamics [15]. By choosing special kernel function J, (1.5) also contains (1.4) as a special case [3]. As for such a pattern in describing the spatial dispersal and the corresponding mathematical theory, we also refer to Coville and Dupaigne [4] for the traveling wave solution of an equation arising in population dynamics, Bates et al. [2] for a convolution model for phase transitions, Bates [1] and Fife [7] for the surveys on nonlocal diffusion models.

To establish the existence of traveling wave fronts of (1.5), which connects 0 with the positive equilibrium state 1, we shall apply the abstract results established in Pan [16]. Concretely, a pair of upper and lower solutions will be given to construct a profile set, in which the existence of traveling wave fronts is proved. In particular, we also show the strict monotonicity of such a traveling wave front.

2. Preliminaries

In this section, we give some necessary preliminary knowledge developed by Pan [16]. We first introduce some notations. Let $C(\mathbb{R},\mathbb{R})$ be the set of the uniform continuous and bounded functions defined on $\mathbb{R}$, which is a Banach space with the supremum norm. Also denote $C_{[0,1]}$ by

$$C_{[0,1]} = \{u(x) : u(x) \in C(\mathbb{R},\mathbb{R}) \text{ and } 0 \leqslant u(x) \leqslant 1,\ x \in \mathbb{R}\}.$$

To describe the results in Pan [16], we first consider the following equation:

$$\frac{\partial u(x,t)}{\partial t} = \int_{\mathbb{R}} J(y)[u(x-y,t) - u(x,t)]dy + f(u(x,t), u(x,t-\tau)), \tag{2.1}$$

in which J and f satisfy the following assumptions:

(J1) $J : \mathbb{R} \to \mathbb{R},\ J(x) = J(-x) \geqslant 0,\ x \in \mathbb{R}$ and $\int_{\mathbb{R}} J(x)dx > 0$;
(J2) for any $\lambda \geqslant 0,\ 0 < \int_{\mathbb{R}} J(x)e^{\lambda x}dx < \infty$;
(f1) $f(0,0) = f(1,1) = 0$;
(f2) $f(u,v)$ is Lipschitz continuous for $u,\ v \in [0,1]$.

We now give the definition of traveling wave solution of (2.1) as follows.

Definition 2.1. A traveling wave solution of (2.1) is a special solution with the form $u(x,t) = \phi(x+ct)$, in which $c > 0$ accounting for the wave speed parameter and $\phi \in C^1(\mathbb{R},\mathbb{R})$ describing the wave profile. Moreover, if $\phi(t)$ is monotone in $t \in \mathbb{R}$, then it is also called a traveling wave front.

By Definition 2.1, let $u(x,t) = \phi(x+ct)$ be a traveling wave solution of (2.1), then it must satisfy the following functional differential equation:

$$c\phi'(t) = \int_{-\infty}^{\infty} J(t-y)[\phi(y) - \phi(t)]dy + f(\phi(t), \phi(t-c\tau)), \quad t \in \mathbb{R}, \tag{2.2}$$

and we are interested in the monotone solution of (2.2) with the following asymptotic boundary conditions (also see the background of traveling wave front by Bates et al. [2] and Coville and Dupaigne [4]):

$$\lim_{t\to-\infty} \phi(t) = 0, \quad \lim_{t\to\infty} \phi(t) = 1. \tag{2.3}$$

By Pan [16, Proposition 4.14], (f2) implies that the following comparison result is true.

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