Optical Fiber Technology 25 (2015) 44–50

Contents lists available at ScienceDirect

Optical Fiber Technology

www.elsevier.com/locate/yofte

Influence of generalized external potentials on nonlinear tunneling of nonautonomous solitons: Soliton management

A. Mahalingam [a], M.S. Mani Rajan [b,*]

[a] Department of Physics, Anna University, Chennai, India
[b] Department of Physics, University College of Engineering, Anna University, Ramanathapuram, India

ARTICLE INFO

Article history:
Received 23 March 2015
Revised 17 July 2015
Available online 1 August 2015

Keywords:
Soliton management
Nonlinear tunneling
Generalized external potentials
Darboux transformation
Generalized nonautonomous NLS equation

ABSTRACT

Soliton control and management using generalized external potentials in an inhomogeneous fiber to the design of high speed optical devices and ultrahigh capacity transmission systems are investigated based on solving the variable-coefficient generalized nonautonomous nonlinear Schrödinger equation with the help of symbolic computation. We construct Lax pair for GNLS equation by means of AKNS method and two soliton solutions are obtained by virtue of the Darboux transformation. With symbolic computation, we manipulate the control parameters and external potentials to investigate the propagation behaviors of nonautonomous solitons. Moreover, the main evolution features of obtained two soliton solutions are exposed by some interesting figures through computer simulation. Especially, we analyze the influence of external potentials such as periodic, exponential and parabolic potential on soliton propagation. Finally, soliton propagation under the absence (vanishing) of external potential is also discussed. Obtained results confirmed that external potentials has strong influence on the soliton dynamics. Our results might provide a new method to achieve the soliton pulse compression while they passing through the potential barrier or well under the influence of external potentials.

1. Introduction

The optical soliton concept grows to be an intensely studied subject with the progress of modern communication technology. Nowadays, it is generally regarded that optical solitons have been playing an important role in fundamental research of nonlinear science and in the field of the next generation optical communication systems with long-haul, high speed and large capacity. Recently, a new concept of soliton control or soliton management has developed in the application of soliton, and has been extensively investigated because of its potential values. Applications of soliton are growing through the whole of physics from hydrodynamics to nonlinear optics, plasma physics, elementary particle physics, etc. Typically, the dynamics of optical soliton in nonlinear optics can be described by a nonlinear Schrodinger equation (NLSE) [1]. In nonlinear optics, the construction of exact solutions for various NLS equations is one of the most important and essential tasks. With the aid of exact solutions of NLS equations, one can realize the phenomena such as the propagation, stability, controlling behavior of the optical soliton. Therefore, solutions of the NLSE are very helpful to understand the nonlinear optics in an optical fiber communication system. It is well known that an ideal optical soliton in fiber, theoretically reported by Hasegawa and Tappert [2,3] and experimentally verified by Mollenauer et al. [4], is based on the exact balance between group velocity dispersion and self-phase modulation. It is very familiar that from the Lax pair, one can construct the multisoliton solution through Darboux transformation.

Classical soliton concept emerges from the constant coefficients of dispersion and nonlinearity in Nonlinear Schrödinger equations. In real applications, this kind of system has no any effects on the control of soliton dynamics. The realization of the first soliton dispersion management experiment in a fiber led to extensive investigation of NLSE with variable coefficients as the governing equations for optical soliton control. It should be noted that in addition to the exact balance between the nonlinear and dispersion effects, the formation of optical solitons in the inhomogeneous fiber is also based on the constrained balance among the variable coefficients. In order to maintain a well-ordered transmission of solitons in a communication system, one usually introduces a slow change of the fiber parameters in the propagation direction. Due to the nonuniformities of the core medium, different effects such as loss (or gain), group velocity dispersion, self-phase modulation, external potential and phase modulation are influenced. Historically, extensive research work has been done for the case

* Corresponding author.
E-mail address: senthilmanirajanofc@gmail.com (M.S. Mani Rajan).

http://dx.doi.org/10.1016/j.yofte.2015.07.013
1068-5200/

when time appears explicitly in the governing nonlinear evolution equation, thereby rendering the system nonautonomous [5,6]. These are more attracted in optical fiber communication technology due to potential applications and its interesting features.

In addition to this, the generalized nonautonomous NLSE model with variable coefficients has more attraction where the soliton evolution control can be achieved by the nonuniform distribution of the nonlinear media. Although such variable coefficients often make the studies hard, it is becoming possible and exercisable for a computer to deal with them with the development of symbolic computation. Nonautonomous soliton solutions are potentially useful for various applications in optical soliton communication system due to their special features. Especially, solitons in nonautonomous physical systems exist only under certain conditions and varying in time nonlinearity and dispersion cannot be chosen independently; they satisfy the exact integrability conditions. This law is called as *S–H* (Serkin–Hasegawa) theorem. This theorem is applied to a nonautonomous system and soliton management are discussed in [6]. The law of soliton adaptation to an external potential has potential applications in nonautonomous soliton. Very recently, nonautonomous solitons in parity time (PT) symmetric potentials for both exponential diffraction decreasing and periodically modulated waveguide has been reported [7]. For the first time, the non-chirped nonautonomous solitons with variable coefficients were considered by Serkin et al. in [8]. Luo et al. suggested that nonautonomous solitons can be controlled by varying dispersion and nonlinearity management [9].

The transformation from a nonautonomous NLSE to a standard autonomous NLSE under some proper conditions is discussed in [10]. Under dispersion and nonlinearity management, both chirp free and chirped nonautonomous solitons have been investigated in [11]. Snake like solitons are observed in graded index grating by Yang et al. [12]. In [13], nonautonomous soliton in both BEC and optical fiber with external potential have been reported in detail. Nonautonomous soliton with external potential have been discussed in [14,15]. Dynamics of dark soliton in Bose–Einstein condensate (BEC) with external potentials is claimed in [16]. Very recently, rogue matter wave propagation with external potential have been reported by Sun et al. [17]. Generalized nonautonomous cubic quintic NLS equation with different types of external potential by means of similarity transformation have been investigated [18]. Dai et al. discussed the Self similar rogue waves of nonautonomous system with linear potential [19]. Very recently, collisions of bright nonautonomous soliton with different form of external potential have been investigated without consider the tunneling behavior [20]. Zhong et al. analyzed the coupled nonlinear Schrödinger equation with external harmonic potential through *F*-expansion technique [21].

Although the nonlinear Schrödinger equation with variable coefficients (NLS-vc) system has been widely investigated, we find that there has not been much discussion about the tunneling of soliton with external potential. An important question is that how the time and spatial dependent external potentials are affecting the tunneling behavior of nonautonomous soliton? How one can control the dynamical behavior of nonautonomous optical soliton accurately through external potentials? These questions are quite meaningful for physicists to study them. Especially in this paper, external potential is phase modulation dependent which is not yet discussed by any authors. Being motivated by the above aspects and to say answers this question, this paper is devoted to the investigation on nonlinear tunneling of nonautonomous soliton in nonlinear optical fibers in the presence of various types of external potentials with phase modulation which is different from earlier studies on the concept of nonautonomous soliton in the fiber.

2. The governing model

Based on the above discussions, we consider the generalized nonautonomous nonlinear Schrödinger equation with both external potential and phase modulation can be written as follows:

$$iQ_z + \frac{D(z)}{2}Q_{tt} + R(z)\,|Q|^2Q + i\Gamma(z)Q - iD(z)\theta(z)Q + V(z,t)Q = 0$$

$$\text{where } \Gamma(z) = \frac{1}{2}\left[\frac{1}{R}\frac{dR}{dz} - \frac{1}{D}\frac{dD}{dz}\right] \text{ and} \tag{1}$$

$$V(z,t) = V_2(z,t) + tV_1(z) + V_0(z)$$

$$V_2(z,t) = t^2\left(\theta^2(z)D(z)R(z) - \frac{1}{2}\frac{d\theta(z)}{dz}\right)$$

Here $Q(z,t)$ is the normalized solitary envelope of optical pulse and z and t denote the normalized distance and time duration of the temporal propagation of the optical soliton in fiber. The functions $D(z)$, $R(z)$, $\Gamma(z)$ and $\theta(z)$ denotes dispersion, nonlinearity, loss or gain and phase modulation respectively. $V(z,t)$ represents the external potential and it is assumed to be a function of propagation distance and time. Especially, $V_1(z)$ and $V_2(z,t)$ are represents the linear and harmonic potential respectively. We stress that external potential term is present in terms of phase modulation in Eq. (1). The study of nonlinear wave propagation in Eq. (1) is of great interest and has wide range of applications. It is not only restricted for optical pulse propagating in inhomogeneous optical fiber media, which has found application in pulse compression, but also for the core of a dispersion-managed soliton. Such combined various external potentials has interesting properties and are especially efficient in supporting matter-wave solitons, such as snakelike soliton, breathing soliton and oscillating solitons. These potentials may be realized in BECs by tuning the external magnetic field and the optically controlled interactions using the FR techniques. By considering these potentials, one can understand many specific features of nonlinear waves including nonautonomous solitons, matter wave solitons, breathing solitons, quasi-breathing solitons and resonance solitons. In this paper, during the whole manipulation process, symbolic computation plays a significant role in the complex calculations. With the development of computer technology, the symbolic computational softwares are more powerful.

The article is organized as follows. In Section 2, we construct the Lax pair through AKNS formalism for Generalized nonautonomous NLSE with variable coefficients. Section 3 describes the arrival of two soliton solutions using Darboux transformation method. Section 4 is devoted to discuss the nonlinear tunneling of bright soliton when it propagates along the fiber with external potential. Section 5 brings our conclusions. These results would stimulate the studies on soliton management in nonautonomous system with external potential.

3. Lax pair

In soliton theory, the Lax pair has importance in that it not only gives a method to solve the initial problem of a given nonlinear equation through the method of inverse scattering, but also plays a vital role in studying the integrable properties of NLS equation such as the Hamiltonian structures, conservation laws etc. And also it is well known that the nonlinear evolution equations in integrable systems all have their corresponding Lax pair representation. The form of Lax pair has deep geometrical and physical significance. From the Lax pair, one can generate analytical soliton solutions conveniently through related Darboux transformation.