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A new fourth-order iterative method for finding multiple roots of nonlinear equations

Li Shengguo [a,*], Liao Xiangke [b], Cheng Lizhi [a,1]

[a] School of Science, National University of Defense Technology, Changsha 410073, China
[b] School of Computer, National University of Defense Technology, Changsha 410073, China

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ABSTRACT

In this paper, we present a new fourth-order method for finding multiple roots of nonlinear equations. It requires one evaluation of the function and two of its first derivative per iteration. Finally, some numerical examples are given to show the performance of the presented method compared with some known third-order methods.

1. Introduction

Finding the roots of nonlinear equations is very important in numerical analysis and has many applications in engineering and other applied sciences. In this paper, we consider iterative methods to find a multiple root α of multiplicity m, i.e., $f^{(j)}(\alpha)=0,\ j=0,1,\ldots,m-1$ and $f^{(m)}(\alpha)\neq 0$, of a nonlinear equation $f(x)=0$.

The modified Newton's method for multiple roots is quadratically convergent and it is written as [1]

$$x_{n+1}=x_n-m\frac{f(x_n)}{f'(x_n)}, \tag{1}$$

which requires the knowledge of the multiplicity m.

In recent years, some modifications of the Newton method for multiple roots have been proposed and analyzed, which also require the knowledge of the multiplicity m. Most of these methods are of third-order convergence. For example, see Traub [2], Hansen and Patrick [3], Victory and Neta [4], Dong [5,6], Osada [7], Neta [8], Chun and Neta [9], Chun et al. [10], etc.

The third-order Chebyshev's method for finding multiple roots [2,8] is given by

$$x_{n+1}=x_n-\frac{m(3-m)}{2}u_n-\frac{m^2}{2}\frac{f(x_n)^2f''(x_n)}{f'(x_n)^3}, \tag{2}$$

where

$$u_n=\frac{f(x_n)}{f'(x_n)}. \tag{3}$$

* Corresponding author.
E-mail addresses: lsg_998@yahoo.com.cn (L. Shengguo), clzcheng@vip.sina.com (C. Lizhi).

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The cubically convergent Halley's method which is a special case of the Hansen and Patrick's method [3], is written as

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{m+1}{2m} f'(x_n) - \frac{f(x_n) f''(x_n)}{2f'(x_n)}}. \tag{4}$$

The third-order Osada method [7], is written as

$$x_{n+1} = x_n - \frac{1}{2} m(m+1) u_n + \frac{1}{2}(m-1)^2 \frac{f'(x_n)}{f''(x_n)}. \tag{5}$$

Dong [5] has developed two third-order methods requiring two evaluations of the function and one of its first derivative

$$\begin{cases} y_n = x_n - \sqrt{m} u_n, \\ x_{n+1} = y_n - m\left(1 - \frac{1}{\sqrt{m}}\right)^{1-m} \frac{f(y_n)}{f'(x_n)}, \end{cases} \tag{6}$$

$$\begin{cases} y_n = x_n - u_n, \\ x_{n+1} = y_n - \frac{f(y_n)}{\left(\frac{m-1}{m}\right)^{m-1} f(x_n) - f(y_n)} u_n. \end{cases} \tag{7}$$

In [11], Neta and Johnson have proposed a fourth-order method requiring one-function and three-derivative evaluation per iteration. This method is based on the Jarratt method [12] given by the iteration function

$$x_{n+1} = x_n - \frac{f(x_n)}{a_1 f'(x_n) + a_2 f'(y_n) + a_3 f'(\eta_n)}, \tag{8}$$

where

$$\begin{cases} y_n = x_n - a u_n, \\ v_n = \frac{f(x_n)}{f'(y_n)}, \\ \eta_n = x_n - b u_n - c v_n. \end{cases} \tag{9}$$

Neta and Johnson [11] give a table of values for the parameters a, b, c, a_1, a_2, a_3 for several values of m. But, they do not give a closed formula for general case.

Neta [13] has developed another fourth-order method requiring one-function and three-derivative evaluation per iteration. This method is based on Murakami's method [14] given by

$$x_{n+1} = x_n - a_1 u_n - a_2 v_n - a_3 w_3 - \psi(x_n), \tag{10}$$

where u_n is defined by (3), v_n, y_n and η_n are given by (9) and

$$\begin{aligned} w_3(x_n) &= \frac{f(x_n)}{f'(\eta_n)}, \\ \psi(x_n) &= \frac{f(x_n)}{b_1 f'(x_n) + b_2 f'(y_n)}. \end{aligned} \tag{11}$$

A table of values for the parameters $a, b, c, a_1, a_2, a_3, b_1, b_2$ for several values of m is also given by Neta [13].

In this paper, we propose a new fourth-order method for multiple roots, which requires one-function and two-derivative evaluation per iteration, while the method proposed in [11] or [13] requires one-function and three-derivative evaluation per iteration. So, the method presented here requires less function evaluations than these methods in [11,13]. Moreover, the presented method here has a closed formula. The presented method is obtained by investigating the following iteration functions

$$\begin{cases} y_n = x_n - \theta u_n, \\ x_{n+1} = x_n - \frac{\beta f'(x_n) + \gamma f'(y_n)}{f'(x_n) + \delta f'(y_n)} u_n, \end{cases} \tag{12}$$

where θ, β, γ and δ are parameters to be determined. When $\theta = \frac{2}{3}, \beta = -\frac{1}{2}, \gamma = -\frac{3}{2}$ and $\delta = -3$, (12) reduces to the fourth-order Jarratt method [15], which is defined by

$$\begin{cases} y_n = x_n - \frac{2}{3} u_n, \\ x_{n+1} = x_n - \left(1 - \frac{3}{2} \frac{f'(y_n) - f'(x_n)}{3f'(y_n) - f'(x_n)}\right) u_n. \end{cases} \tag{13}$$

By specially choosing the parameters, we get a new fourth-order method based on (12) in Section 2. Analysis of convergence shows that the new method has fourth-order convergence. Finally, we use some numerical examples to compare the new fourth-order method with some known third-order methods. From the results, we can see that the fourth-order method converges faster than these third-order methods and requires less function evaluations.