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Exponential stability criterion for time-delay systems with nonlinear uncertainties

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ABSTRACT

Exponential stability of time-delay systems with nonlinear uncertainties is studied in this paper. Based on the Lyapunov method and the approaches of decomposing the matrix, a new exponential stability criterion is derived in terms of a matrix inequality, which allows to compute simultaneously the two bounds that characterize the exponential nature of the solution. Some numerical examples are also given to show the superiority of our result to those in the literature.

1. Introduction

Consider the following time-delay systems with nonlinear uncertainties:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t-h) + f(t,x(t)) + f_1(t,x(t-h)), \\ x_0(\theta) = \phi(\theta), \end{cases} \tag{1.1}$$

where $x(t) \in R^n$ is the state, A, A_1 are given matrix, and initial condition is $x_0(\theta) = \phi(\theta) \in C([-h,0],\mathbb{R}^n)$. The time-varying parameter uncertainties f, f_1 are assumed to be bounded

$$\|f(t,x(t))\| \leqslant \alpha\|x(t)\|, \quad \|f_1(t,x(t-h))\| \leqslant \alpha_1\|x(t-h)\|,$$

where α, α_1 are positive numbers.

Definition 1.1. The system (1.1) is δ-stable, with $\delta > 0$, if there is a positive number N such that for each $\phi(.)$, the solution $x(t,\phi)$ of the system (1.1) satisfies

$$\|x(t,\phi)\| \leqslant Ne^{-\delta t}\|\phi\| \quad \forall t \geqslant 0,$$

where $\|\phi\| = \max\{\|\phi(t)\| : t \in [-h,0]\}$. N is called Lyapunov factor.

Because of data errors, environmental noises, the difficulty of measuring various parameters, unavoidable approximation, etc., most real problems are modeled by time-delay systems with nonlinear uncertainties. So, the stability problem of time-delay system with nonlinear uncertainties has been an interesting problem in the recent years (see in [2–12] and reference therein). It is well known that the widely used method is the approach of Lyapunov functions with Razumikhin techniques and the stability conditions are presented in terms of the solution of either linear matrix inequalities or Riccati equations [5,8,9]. By using parameterized neutral models, some less conservative criteria, which are dependent on the stability of the operator, have proposed in [2,10–12]. By proposing a technique to adjust the Lyapunov functionals in [2,11,12], the

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authors in [6,7] have reduced the stability of the operator and given some less conservative more criteria. However, this technique has not applied to the Lyapunov functional in [10]. Therefore, the first purpose of this paper is to find an improvement the criterion in [10] by using this technique.

Inspired by the method of decomposing the matrix in [1], we have decomposed the matrix A_1 into two parts A_{11}, A_{12}. Using the operator

$$D(x_t) = x(t) - A_{11}\int_{t-h}^{t} x(s)ds, \quad (1.2)$$

we get a generalization of the result in [10]. Combining an exponential translation variable and the technique to reduced the stability of the operator, we get a new δ-stability criterion for the system (1.1). As a consequence of the this criterion, we also obtain an asymptotical stability criterion. Compared with the results in [10], our result has the following advantages:

- First, by decomposing the matrix A_1 into two part A_{11}, A_{12} and using the operator $D(x_t) = x(t) - A_{11}\int_{t-h}^{t} x(s)ds$, our criterion will be less restricted than the criterion in [10].
- Second, by reducing the stability of the operator, our criterion will be less conservative than the criterion in [10].

The following lemma is needed for our main results.

Lemma 1.1 [6]. *Assume that $S \in \mathbb{R}^{n\times n}$ is a symmetric positive-definite matrix. Then for every $Q \in \mathbb{R}^{n\times n}$,*

$$2\langle Qy, x\rangle - \langle Sy, y\rangle \leqslant \langle QS^{-1}Q^T x, x\rangle \quad \forall x, y \in \mathbb{R}^n.$$

If we take $S = I$ then we have $|2\langle Qy, x\rangle| \leqslant \|y\|^2 + \|Qx\|^2$. By decomposing the matrix A_1 into two parts A_{11}, A_{12} and using the operator $D(x_t) = x(t) - A_{11}\int_{t-h}^{t} x(s)ds$, we get a generalization of the result in [10]. This generalization also need for our main results.

Theorem 1.2. *For given $h > 0$ and α, α_1, the system (1.1) is asymptotically stable if the exist the positive-definite matrices X, Z_1, Z_2, M and positive scalars ϵ_0, ϵ_1 and $0 < \beta < 1$ satisfying the following two matrix inequalities:*

$$\begin{pmatrix} -\beta M & hA_{11}^T M \\ \star & M \end{pmatrix} < 0, \quad (1.3)$$

$$\Sigma_0 < 0, \quad (1.4)$$

where

$$\Sigma_0 = \begin{pmatrix} \Xi_{11} & I & I & \Xi_{14} & \Xi_{15} & 0 & hX & \epsilon_0\alpha X & X \\ \star & -\epsilon_0 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -h^{-1}Z_1 & 0 & 0 & -hZ_1A_{11}^T & -\epsilon_0\alpha Z_1 A_{11}^T & -Z_1A_{11}^T \\ \star & \star & \star & \star & -Z_2 & \epsilon_1\alpha_1 Z_2 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\epsilon_1 I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -hZ_1 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\epsilon_0 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -Z_2 \end{pmatrix}$$

with

$$\Xi_{11} = (A + A_{11})X + X(A + A_{11})^T,$$
$$\Xi_{14} = -(A + A_{11})A_{11}Z_1,$$
$$\Xi_{15} = (A_1 - A_{11})Z_2.$$

Proof. Consider the following Lyapunov functional:

$$V = V_1 + V_2 + V_3, \quad (1.5)$$

where

$$V_1 = \int_{t-h}^{t} (s - t + h)x^T(s)R_1x(s)ds,$$
$$V_2 = \int_{t-h}^{t} x^T(s)R_2x(s), \quad V_3 = \mathscr{D}^T(x_t)P\mathscr{D}(x_t).$$