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A superlinearly convergent norm-relaxed SQP method of strongly sub-feasible directions for constrained optimization without strict complementarity ☆

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ABSTRACT

In this paper, a kind of optimization problems with nonlinear inequality constraints is discussed. Combined the ideas of norm-relaxed SQP method and strongly sub-feasible direction method as well as a pivoting operation, a new fast algorithm with arbitrary initial point for the discussed problem is presented. At each iteration of the algorithm, an improved direction is obtained by solving only one direction finding subproblem which possesses small scale and always has an optimal solution, and to avoid the Maratos effect, another correction direction is yielded by a simple explicit formula. Since the line search technique can automatically combine the initialization and optimization processes, after finite iterations, the iteration points always get into the feasible set. The proposed algorithm is proved to be globally convergent and superlinearly convergent under mild conditions without the strict complementarity. Finally, some numerical tests are reported.

1. Introduction

In this work, we consider the nonlinear inequality constrained optimization problem:

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_j(x) \leqslant 0, \quad j \in I = \{1, 2, \ldots, m\}, \end{array} \tag{1.1}$$

where the functions $f,\ g_j\ (j \in I) : R^n \to R$ are all smooth. We denote the feasible set of the problem (1.1) by $X = \{x \in R^n : g_j(x) \leqslant 0, j \in I\}$.

It is well known that the sequential quadratic programming (SQP) algorithm is one of the most effective algorithms available for solving (1.1) because of its superlinear convergence (see e.g. [1–4]). However, the solution of the direction finding subproblem (DFS), i.e., quadratic program (QP) is not a feasible direction in general, and can not avoid the Maratos effect. Therefore, it must be updated by some suitable techniques. Generally, these updated techniques increase the computational cost and complexity of the algorithms, which will make it necessary to solve two or three QPs per single iteration, such as

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[1,3], moreover, the superlinear convergence of most of these algorithms depends on the strict complentarity condition, such as [1,3,4], which is rather strong and difficult for checking.

The method of feasible directions (MFD) is also a classical method for solving the problem (1.1). The theoretical basis for MFD was originally developed by Zoutendijk [5]. Since the MFD possesses many advantages, e.g., the iterative points and the approximate optimal solutions are all feasible, which property is very important in some engineering designs, the sequence $\{f(x^k)\}$ of objective function is decreasing and so on, the details can be seen in [6], many researches on MFD have been done (see e.g. [7–14]). In 1994, Cawood and Kostreva [7] proposed a norm-relaxed MFD, then a generalized norm-relaxed MFD was presented by Chen and Kostreva [8] in 1999, where more parameters was introduced to improve the numerical results and speed up the convergent rate. The DFS in [8] has the form of

$$\begin{aligned} \min \quad & z+\tfrac{1}{2}d^TB_kd \\ \text{s.t.} \quad & \nabla f(x^k)^Td \leqslant r_0z, \\ & g_j(x^k)+\nabla g_j(x^k)^Td \leqslant r_jz, \quad j\in I, \end{aligned}$$

where B_k is a positive definite matrix and γ_0, γ_j $(j \in I)$ are all positive constants. To achieve superlinear convergence, the generalized norm-relaxed MFD was further studied by Kostreva and Chen [9], Lawrence and Tits [10] as well as Zhu and Zhang [11], etc., in these algorithms, a high-order correction direction is yielded by solving another QP subproblem [9,10] or by an explicit formula [11]. However, the superlinear convergence property of these methods also needs the strict complementarity assumption. In addition, these algorithms have a common shortcoming that the initial point must be feasible, so an auxiliary procedure must be considered for finding an initial feasible point.

In 1979, Polak et al. [12] proposed a combined Phase I–Phase II algorithm with arbitrary initial point. The algorithm become a MFD when the iteration point gets into the feasible region. Jian [13] improved the algorithm and presented a method of strongly sub-feasible method in 1995, which not only unified automatically the processes of initialization (Phase I) and optimization (Phase II), but also guaranteed that the number of the functions satisfying inequality constraints is nondecreasing. In 2005, based on the idea of the strongly sub-feasible direction method and Norm-Relaxed method, Jian et al. [14] proposed a globally convergent norm-relaxed method of strongly sub-feasible directions, the associated DFS corresponding the iterative x^k has the following form:

$$\begin{aligned} \min \quad & z+\tfrac{1}{2}d^TB_kd \\ \text{s.t.} \quad & \nabla f(x^k)^Td \leqslant \gamma_0z+\alpha(x^k), \\ & g_j(x^k)+\nabla g_j(x^k)^Td \leqslant \gamma_j\eta_kz, \quad j\in I^-(x^k), \\ & g_j(x^k)+\nabla g_j(x^k)^Td \leqslant \gamma_j\eta_kz+\varphi(x^k), \quad j\in I^+(x^k), \end{aligned}$$

where $I^-(x^k)=\{j\in I: g_j(x^k)\leqslant 0\},\ I^+(x^k)=\{j\in I: g_j(x^k)>0\}$, $\alpha(x^k)$ is a penalty function, η_k is a positive parameter associated with x^k, γ_j $(j=0,1,\dots,m)$ are all positive constant parameters and $\varphi(x^k)$ is the maximal violated constraint function. However, as no high-order correction direction is introduced and computed, the algorithm in [14] only possesses global convergence. In 2006, by introducing a generalized projection high-order correction direction, the authors [15] further modified the algorithm [14] such that it has superlinear convergence, where all gradients of the constraint functions are used in the generalized projection. Obviously, it can be seen that the constraints of the DFS [14,15] include all the constraints of the original problem (1.1) and the high-order correction directions are yielded by all gradients, all these greatly add the scale and computational cost of the algorithms.

In this paper, by introducing a new pivoting operation and another explicit high-order correction direction, we modified the algorithms in [14,15] by another approach, as a result, a new norm-relaxed SQP algorithm is presented. Moreover, at the end of this paper, we test our algorithm by some practice problems. The main features of the proposed algorithm are as follows:

- the initial point is arbitrary;
- an improved direction for the problem (1.1) is obtained by solving only one sub-problem whose constraints only include an estimate of the active constraints. Therefore, compared to Ref. [14,15], the scale and the computation cost of the subproblem are further reduced;
- the number of constraints satisfying constraint condition is nondecreasing;
- after finite number of iterations, the iteration points always get into the feasible set X;
- a high-order updated direction which can avoid the Maratos effect is yielded by a usual projection formula which need only the gradients associated with an ε-active constraint set;
- it possesses global convergence and superlinear convergence under some mild assumptions without the strictly complementarity, especially, the algorithm itself does not relay on the linearly independent constraint qualification (LICQ).