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An iterative method with quartic convergence for solving nonlinear equations

Rostam K. Saeed *, Kawa M. Aziz

University of Salahaddin/Erbil, College of Science, Department of Mathematics, Kurdistan Region, Iraq

Abstract

In this study, a new two-step and three-step iterative methods are derived for solving nonlinear equations. It is shown that the three-step iterative method has fourth-order convergence. Several numerical examples are given to illustrate the efficiency and performance of the new method.
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1. Introduction

There are different methods to approximate the solution of nonlinear equation, such as the Newton–Raphson method with quadratic convergence. Recently, Abbasbandy [1] modified Newton–Raphson method for solving the nonlinear algebraic equation $f(x) = 0$, by considering terms up to second-order in the Taylor series and using Adomain decomposition method. Further, Noor et al. [2] considered terms up to second-order in Taylor series but used the new iterative method proposed by Daftardar-Gejji and Jafari [3]. In this work, two iterative methods were constructed by considering terms up to fourth-order in Taylor series and the new iterative methods proposed by Noor et al. [2] and Daftardar-Gejji and Jafari [3].

2. Iterative methods

Consider the nonlinear algebraic equation

$$f(x) = 0, \tag{1}$$

where α be a root of it and f is a C^4 function on an interval containing α. Using fourth-order Taylor series, the nonlinear equation (1) can be written as follows:

* Corresponding author.
E-mail addresses: rostamkarim64@uni-sci.org (R.K. Saeed), kawama2005@yahoo.com (K.M. Aziz).

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$$f(\gamma) + (x-\gamma)f'(\gamma) + \frac{(x-\gamma)^2}{2!}f''(\gamma) + \frac{(x-\gamma)^3}{3!}f'''(\gamma) + \frac{(x-\gamma)^4}{4!}f^{(4)}(\gamma) = 0, \tag{2}$$

where γ is the initial approximation for a zero of Eq. (1).

In a similar manner of Noor et al. [2], to construct two types of iterative methods, Eq. (2) can be rewritten as follows:

$$x = c + N(x), \tag{3}$$

where

$$c = \gamma - \frac{f(\gamma)}{f'(\gamma)} \tag{4}$$

and

$$N(x) = -\frac{(x-\gamma)^2}{2f'(\gamma)}f''(\gamma) - \frac{(x-\gamma)^3}{3!f'(\gamma)}f'''(\gamma) - \frac{(x-\gamma)^4}{4!f'(\gamma)}f^{(4)}(\gamma). \tag{5}$$

To complete the derivation of the iterative methods, we use the decomposition scheme of Daftardar-Gejji and Jafari [3], which is given below.

Suppose a solution of Eq. (3) having the series form:

$$x = \sum_{i=0}^{\infty} x_i. \tag{6}$$

The nonlinear operator N can be decomposed as

$$N\left(\sum_{i=0}^{\infty} x_i\right) = N(x_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right)\right\}. \tag{7}$$

From Eqs. (3), (6) and (7), we get

$$x = \sum_{i=0}^{\infty} x_i = c + N(x_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} x_j\right) - N\left(\sum_{j=0}^{i-1} x_j\right)\right\}. \tag{8}$$

We define the recurrence relation:

$$\begin{aligned}
x_0 &= c,\\
x_1 &= N(x_0),\\
x_2 &= N(x_0+x_1) - N(x_0),\\
&\vdots\\
x_n &= N(x_0+x_1+\cdots+x_{n-1}) - N(x_0+x_1+\cdots+x_{n-2}), \quad n = 1,2,\ldots.
\end{aligned} \tag{9}$$

Then

$$X_n = x_0 + x_1 + x_2 + \cdots + x_n = N(x_0+x_1+\cdots+x_{n-1}), \quad n = 1,2,\ldots \tag{10}$$

Denotes the $(n+1)$-term approximation of x, see [1].

From Eq. (10), for $n = 1$ and Eq. (5), we get

$$\begin{aligned}
x \approx X_1 = x_0 + x_1 &= c + N(x_0)\\
&= \gamma - \frac{f(\gamma)}{f'(\gamma)} + \left[-\frac{(x_0-\gamma)^2}{2f'(\gamma)}f''(\gamma) - \frac{(x_0-\gamma)^3}{3!f'(\gamma)}f'''(\gamma) - \frac{(x_0-\gamma)^4}{4!f'(\gamma)}f^{(4)}(\gamma)\right]\\
&= \gamma - \left[\frac{f(\gamma)}{f'(\gamma)} + \frac{(x_0-\gamma)^2}{2f'(\gamma)}f''(\gamma) + \frac{(x_0-\gamma)^3}{3!f'(\gamma)}f'''(\gamma) + \frac{(x_0-\gamma)^4}{4!f'(\gamma)}f^{(4)}(\gamma)\right].
\end{aligned}$$

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