Applied Mathematics and Computation 214 (2009) 60–65

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Oblique interactions between multi-solitons in plasma

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ARTICLE INFO

Keywords:
Soliton interaction
Phase shift

ABSTRACT

The nonlinear waves in two-dimensional plasma are studied in this paper. Solitary waves propagating in two different directions are investigated as well. The equation of motion of nonlinear wave in the different directions is also obtained. The phase shifts after the interaction for multi-solitons in different directions are given in the present work.

1. Introduction

Studies on wave propagation in plasmas have provided one of the keystones in the development of plasma physics and they remain a focus in contemporary research [1]. Plasma waves whether in laboratory and in space are in general nonlinear [2,3]. Moreover, real plasmas are at the same time inhomogeneous, anisotropic, dissipative and dispersive [4–6]. To avoid being overwhelmed by detail at the outset some radical simplifications are needed and so we begin by assuming the medium is unbounded. Based on these assumptions a large number of plasma waves have been studied recently, such as ion-acoustic waves [7], dust-acoustic waves [8–11] and dust–ion-acoustic waves [12,13]. Furthermore, solitons and their interactions have also been found nearly in all of these nonlinear plasma waves [14,15]. Many researchers are interested in studying solitons in plasma physics and in nearly all branches of physics [16–24].

A general formulation of the problem of nonlinear wave propagation via fundamental sets of equations, such as, the Maxwell or Navier–Stokes equations, is a very demanding task even for modern computer. Therefore, a number of simplified models have been introduced which approximately describe either propagation of the waves itself, e.g., the Korteweg de Vries (KdV) equation, or propagation of a slowly varying wave envelopes, e.g., the cubic nonlinear Schrodinger (NLS) equation [24].

The interactions between two KdV solitons have been well studied in one-dimensional (1D) system [24]. There are two distinct soliton interactions for 1D system. One is overtaking collision and the other is the head-on collision. The overtaking collision of the solitons can be studied with the KdV equation. Its multi-soliton solutions traveling in same direction can be obtained from the inverse scattering transform [24]. For the head-on collision, we can obtain the solutions by employing the suitable asymptotic expansion to solve the original equations [24,25]. However, in two-dimensional (2D) system both the overtaking collision and head-on collision are only two special cases of $\alpha = 0$ and $\alpha = \pi$, respectively, where α is the angle between two propagation directions of two solitons. In order to study the general case of $0 \leqslant \alpha \leqslant \pi$ in 2D system, in this paper, we choose the ion acoustic waves in plasmas as an example. Moreover, we can obtain their phase shift after collision of two solitons. Our approach of investigations of two soliton interaction with an arbitrary propagation angle can be extended in any branches of physics to study interactions of multi-solitons propagating in arbitrary directions in two-dimensional or three-dimensional systems.

We obtain the equations of motion for nonlinear waves in two different directions in Section 2. We study the solitary waves and their phase shifts after interaction for one soliton solution, two soliton solution and three soliton solution in Section 3. In Section 4, we similarly give the results for N solitons and some conclusions.

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doi:10.1016/j.amc.2009.03.063

2. Equation of motion

The dimensionless 2D basic equations of ion acoustic waves in a plasma are as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}+\frac{\partial (nv)}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial \phi}{\partial y}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=e^{\phi}-n, \tag{4}$$

where n is the normalized number densities of ions, ϕ is the normalized electrostatic potential, u and v are the normalized velocities of ions in the x and y directions, respectively. The ion number density n is normalized by n_0, the ion number density at equilibrium. ϕ is normalized by eL/kT_e. The space coordinates x and y, time t and the velocity of u and v are normalized by L, ω_0^{-1}, and $L\omega_0$, respectively, where $L=\left(\frac{kT_e}{4\pi Ze^2 n_0}\right)^{1/2}$, $\omega_0=\left(\frac{4\pi n_0 e^2 Z^2}{m_i}\right)^{1/2}$.

There are a lot of expansion methods successfully used in many branches of physics to find the solutions of complex systems, which usually can be described by sets of partial differential equations. One of them is Poincaré–Lighthill–Kuo (PLK) method. It has been used to study the head-on collision between two solitons in 1D system. The head-on collision can results in a change of trajectory or phase shift for each soliton. For 2D systems it is anticipated that the interaction between two solitons can also result in a phase shift for each soliton, and this phase shift should depend on the angle α between two solitons, or it is a function of angle α. In order to find the dependence of phase shift on the α and to study the interaction between two solitary waves with an arbitrary angle between both propagation directions in 2D system, we extend the reductive perturbation method to 2D system. We now search for the evolution waves traveling in two different propagation directions. Therefore, we need to employ a suitable asymptotic expansion to solve the original equations of motion. Starting from Eq. (1)–(4), we assume that in the plasma, solitary waves propagating in two different directions are generated. As the time increase, these solitary waves interact, collide and then depart. Hence we use the following asymptotic expansions:

$$\begin{pmatrix} n \\ u \\ v \\ \phi \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}+\epsilon^2\begin{pmatrix} n_1 \\ u_1 \\ v_1 \\ \phi_1 \end{pmatrix}+\epsilon^4\begin{pmatrix} n_2 \\ u_2 \\ v_2 \\ \phi_2 \end{pmatrix}+\cdots, \tag{5}$$

where ϵ is a small parameter which characterize the relative amplitude of the excitation, we assume that n, u, v and ϕ are functions of multiple-scale variables defined by

$$\begin{pmatrix} \xi \\ \eta \\ \tau \end{pmatrix}=\begin{pmatrix} \epsilon(k_1x+l_1y-c_\xi t)+\epsilon^2P^{(0)}(\eta,\tau)+\cdots \\ \epsilon(k_2x+l_2y-c_\eta t)+\epsilon^2Q^{(0)}(\xi,\tau)+\cdots \\ \epsilon^3 t \end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} c_\xi \\ c_\eta \end{pmatrix}=\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \tag{7}$$

which denote the trajectories of the solitary waves traveling in two different directions, respectively. The functions of $P^{(0)}(\eta,\tau)$ and $Q^{(0)}(\xi,\tau)$ are to be determined. The aim of introducing these functions is to make a uniformly valid asymptotic expansion (i.e., to eliminate the secular terms) and at the same time to obtain the values of phase shifts of the solitary waves after collision.

It follows from Eq. (6) that two waves travel in two different directions of $\vec{q_1}=(k_1,l_1)$ and $\vec{q_2}=(k_2,l_2)$, respectively. The angle α between two wave propagation directions can be given by $\cos\alpha=\frac{k_1k_2+l_1l_2}{\sqrt{k_1^2+l_1^2}\sqrt{k_2^2+k_2^2}}$. Here we must know that $\alpha\neq 0$ or $\cos\alpha\neq 1$. Otherwise, our perturbation method in this paper are not valid. Substituting Eqs. (5)–(7) into Eqs. (1)–(4), we obtain, at the lowest order of ϵ, the following equations: $n_1=n_\xi(\xi,\tau)+n_\eta(\eta,\tau)$, $u_1=u_\xi(\xi,\tau)+u_\eta(\eta,\tau)$, $v_1=v_\xi(\xi,\tau)+v_\eta(\eta,\tau)$, $\phi_1=\phi_\xi(\xi,\tau)+\phi_\eta(\eta,\tau)$, $c_1^2=k_1^2+l_1^2, c_2^2=k_2^2+l_2^2$, $u_\xi=\frac{k_1}{c_1}n_\xi$, $v_\xi=\frac{l_1}{c_1}n_\xi$, $u_\eta=\frac{k_2}{c_2}n_\eta$, $v_\eta=\frac{l_2}{c_2}n_\eta$, $\phi_\xi=n_\xi$ and $\phi_\eta=n_\eta$.

At the next order of ϵ, we obtain, after tedious calculations, the following equations:

$$\frac{\partial n_\xi}{\partial \tau}+\frac{c_1}{2}n_\xi\frac{\partial n_\xi}{\partial \xi}+\frac{c_1^3}{2}\frac{\partial^3 n_\xi}{\partial \xi^3}=0. \tag{8}$$