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Extensions of Faddeev's algorithms to polynomial matrices ☆

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ABSTRACT

Starting from algorithms introduced in [Ky M. Vu, An extension of the Faddeev's algorithms, in: Proceedings of the IEEE Multi-conference on Systems and Control on September 3–5th, 2008, San Antonio, TX] which are applicable to one-variable regular polynomial matrices, we introduce two dual extensions of the Faddeev's algorithm to one-variable rectangular or singular matrices. Corresponding algorithms for symbolic computing the Drazin and the Moore–Penrose inverse are introduced. These algorithms are alternative with respect to previous representations of the Moore–Penrose and the Drazin inverse of one-variable polynomial matrices based on the Leverrier–Faddeev's algorithm. Complexity analysis is performed. Algorithms are implemented in the symbolic computational package MATHEMATICA and illustrative test examples are presented.

1. Introduction

As usual, by $\mathbb{R}^{m\times n}$ we denote the set of $m\times n$ complex matrices. Similarly, $\mathbb{R}[s]$ (resp. $\mathbb{R}(s)$) denotes the polynomials (resp. rational functions) with real coefficients in the indeterminate s. The set of $m\times n$ matrices with elements in $\mathbb{R}[s]$ (resp. $\mathbb{R}(s)$) is denoted by $\mathbb{R}[s]^{m\times n}$ (resp. $\mathbb{R}(s)^{m\times n}$). By I is denoted an appropriate identity matrix, $\mathbb{O}$ denotes zero matrix of adequate dimensions and by $\mathbf{0}$ is denoted the zero polynomial. The trace of given square matrix is denoted by $\mathrm{Tr}(A)$.

For any matrix $A\in\mathbb{C}^{m\times n}$, the following system of matrix equations

$$(1)\ AXA=A,\quad (2)\ XAX=X,\quad (3)\ (AX)^T=AX,\quad (4)\ (XA)^T=XA$$

has unique solution with respect to matrix $X\in\mathbb{R}^{n\times m}$, known as the Moore–Penrose generalized inverse of matrix A and denoted by $A^\dagger$.

Let $A\in\mathbb{R}^{n\times n}$ be arbitrary matrix and let $k=\mathrm{ind}(A)$. Then the following system of matrix equations

$$(1^k)\ A^kXA=A^k,\quad (2)\ XAX=X,\quad (5)\ AX=XA$$

has unique solution. This solution is called the Drazin inverse of matrix A and denoted by A^D.

The algorithms to calculate the determinant and adjoint polynomials of the matrix inverse $(sI-A)^{-1}$, known as the resolvent of A, are discussed in [3,4,9,25], for example. In Kailath [9], the author gave corresponding algorithms by calling underlying formulas as the Leverrier–Souriau–Faddeeva–Frame formulas.

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An extension of the Leverrier–Faddeev algorithm which computes the Moore–Penrose inverse of the constant rectangular matrix $A \in \mathbb{C}^{m\times n}$ is given in [2]. An analogous algorithm for computing the Drazin inverse of a constant, square, possibly singular matrix $A \in \mathbb{C}^{n\times n}$ is introduced in [6].

Computation of the Moore–Penrose inverse of one-variable polynomial and/or rational matrices, based on the Leverrier–Faddeev algorithm, is investigated in [5,8,11,15,23]. Implementation of this algorithm, in the symbolic computational language MAPLE, is described in [8]. Algorithm for computing the Moore–Penrose inverse of two-variable rational and polynomial matrix is introduced in [16]. An effective (quicker and less memory-expensive) algorithm for computing the Moore–Penrose inverse of one-variable and two-variable sparse polynomial matrix, with respect to those introduced in [11], is presented in [13]. This algorithm is efficient when elements of the input matrix are sparse polynomials with only few nonzero addends.

Representations and corresponding algorithms for computing the Drazin inverse of a nonregular polynomial matrix of an arbitrary degree is introduced in [7,21,23]. These algorithms are also extensions of the Leverrier–Faddeev algorithm. Bu and Wei in [1] proposed a finite algorithm for symbolic computation of the Drazin inverse of two-variable rational and polynomial matrices. Also, a more effective three-dimensional version of this algorithm is presented in the paper [1]. Implementation of these algorithms in the programming language MATLAB is also presented in [1].

The algorithm introduced in [22] generalizes the Leverrier–Faddeev algorithm and generates the class of outer inverses of a rational or polynomial matrix.

An interpolation algorithm for computing the Moore–Penrose inverse of a given one-variable polynomial matrix, based on the Leverrier–Faddeev method, is presented in [17]. Corresponding algorithms based on the interpolation and Leverrier–Faddeev algorithms, for computing the Drazin inverse and outer inverses of one-variable polynomial matrix, are introduced in [18,19], respectively. Algorithms for computing the Moore–Penrose and the Drazin inverse of one-variable polynomial matrices based on the evaluation–interpolation technique and the discrete Fourier transform (DFT) are introduced in [14]. Corresponding algorithms for two-variable polynomial matrices are introduced in [24].

We are directly motivated by an (independent) approach for computing the usual inverse, which also starts from the Leverrier–Faddeev's algorithm, but it is applicable to square invertible one-variable polynomial matrices. This approach is initiated by Vu in the papers [26,27]. This approach uses derivative of the matrix powers.

Guided by this motivation, we are going to accomplish the following goals:

1. To extend algorithms introduced in [26,27] for the set of rectangular or singular polynomial matrices. In this way we derive two similar algorithms for computing the Moore–Penrose and the Drazin inverse, respectively;
2. To compare computational complexity and memory space requirements of two different approaches.

In the present paper we will derive an algorithm to calculate the Moore–Penrose and an analogous representation of the Drazin inverse of one-variable polynomial matrix. These algorithms are alternative to known algorithms for computing the Moore–Penrose inverse [8,10,11,15] and the Drazin inverse of polynomial matrices [7,12,21,23]. On the other side, these algorithms generalize algorithms for computing the usual inverse of polynomial matrices, introduced in [26,27].

The paper is organized as follows. The Faddeev's algorithms for computing the Moore–Penrose inverse and the Drazin inverse of rational matrices are reviewed in Section 2. The extension algorithms to one-variable rectangular or singular polynomial matrices are derived in Section 3. Two similar algorithm for computing the Drazin inverse and the Moore–Penrose inverse of polynomial matrices are introduced. Therefore, we implemented our first goal in the third section. In Section 4 we examine complexity analysis of known and introduced algorithms. A comparison between the complexity of introduced and known algorithms is presented. In this way, we implemented our second goal in the fourth section. Some test examples from [30] are verified in Section 5 to verify additionally correctness of introduced algorithms.

2. Faddeev's algorithms for rational matrices

Consider a square matrix constant $A \in \mathbb{R}^{n\times n}$. Assume that the characteristic polynomial of A is equal to

$$a(z) = \det[zI_n - A] = a_0 z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n, \quad a_0 = 1.$$

Representation of the Drazin inverse A^D which is based on the usage of the characteristic polynomial $a(z)$ of the matrix A is introduced in [6].

The following representation of the Drazin inverse is valid for both rational and polynomial square matrices [7,12,21,23] and it is derived as a natural extension of the corresponding representation from [6], applicable to constant square matrices.

Lemma 2.1. *Consider a nonregular one-variable $n \times n$ rational matrix $A(s)$. Assume that*

$$a(z,s) = \det[zI_n - A(s)] = a_0(A)z^n + a_1(A)z^{n-1} + \cdots a_{n-1}(A)z + a_n(A), \quad a_0(A) \equiv 1,\ a_i(A) \in \mathbb{R}[s],\ z \in \mathbb{R} \tag{2.1}$$

is the characteristic polynomial of $A(s)$. Also, consider the following sequence of $n \times n$ polynomial matrices

$$B_i(A) = a_0(A)A(s)^i + a_1(A)A(s)^{i-1} + \cdots a_{i-1}(A)A(s) + a_i(s)I_n, \quad a_0(A) = 1,\ i = 0,\ldots,n. \tag{2.2}$$