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A wavelet method for solving singular integral equation of MHD

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ABSTRACT

This paper presents Haar wavelet approximation to solve a singular integral equation which has singularities on a diagonal of the domain $R = [-1, 1] \times [-1, 1]$. The singularities arise basically due to modified Bessel function K_0 which appears as a part of the kernel. Thus the integral equation is weakly (logarithmically) singular only. The problem is solved considering all the singularities of the kernel and results are examined for approximations of different orders. Our interest to solve the problem using Haar wavelet basis is due to its simplicity and efficiency in numerical approximation. The results show convergence trend as mesh is refined. Comparison is made with some available results obtained earlier by partial consideration of the singularities.

1. Introduction

The present paper deals with the numerical solution of an integral equation which is encountered while modeling the flow of a conducting fluid under the influence of a transverse magnetic field. The analytical solution of the problem was given by Grinberg [1] in terms of Green's function. After satisfying the boundary conditions the final result boils down to solving an integral equation which has singularities on the entire line in the domain of interest. An effort was made to obtain the numerical solution by Singh and Agarwal [2] by taking into account some of singularities and ignoring the rest. In the recent years, interest has grown in applying wavelet approximations to the unknown functions [3–5]. The main reason appears to be simplicity and therefore, the efficiency in the numerical computation. Particularly, the Haar wavelet basis leads to very simple algorithms which converge as the order of approximation is increased. We have tried it for the above problem by taking into account all the singularities of the kernel and computing results for approximations of various orders.

2. Basic equations and analytical solution

The basic equations of magnetohydrodynamic (MHD) flow problem in a straight channel of square section can be put in the following nondimensional form [6,7]

$$V_{xx} + V_{yy} + MB_x = -1, \tag{1}$$
$$B_{xx} + B_{yy} + MV_x = 0, \tag{2}$$

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in the domain $R = [-1,1] \times [-1,1]$ with the following boundary conditions

$$V = 0, B = 0 \quad \text{on} \quad x = \pm 1, \tag{3}$$
$$V = 0, B_y = 0 \quad \text{on} \quad y = \pm 1. \tag{4}$$

Physically, $V(x,y)$ is the velocity and $B(x,y)$ is the induced magnetic field. The parameter M is called Hartmann number. The geometry of the problem is explained in Fig. 1 where the walls $x = \pm 1$ are shown nonconducting while $y = \pm 1$ as perfectly conducting. Further details about the physics of the problem are already available in the literature on MHD [2,6,7].

For us Eqs. (1)–(4) comprise a boundary value problem which we wish to solve using wavelet approximation. As explained in [2] the Eqs. (1) and (2) can be decoupled by the substitution

$$V + B = e^{-\alpha x}(u + p) - x/2\alpha, \tag{5}$$
$$V - B = e^{\alpha x}(v + q) + x/2\alpha, \tag{6}$$
$$\text{with} \quad p(x,\alpha) = (e^{\alpha x} ch2\alpha - e^{-\alpha x})/(2\alpha sh2\alpha), \tag{7}$$
$$q(x,\alpha) = (e^{-\alpha x} ch2\alpha - e^{\alpha x})/(2\alpha sh2\alpha), \tag{8}$$
$$\alpha = M/2. \tag{9}$$

This changes the BVP to

$$u_{xx} + u_{yy} = \alpha^2 u, \tag{10}$$
$$\text{and} \quad v_{xx} + v_{yy} = \alpha^2 v, \tag{11}$$

with boundary conditions

$$u = 0, \quad v = 0 \quad \text{on } x = \pm 1, \tag{12}$$
$$u_y = 0, \quad v_y = 0 \quad \text{on } y = 0, \tag{13}$$
$$e^{-\alpha x} u_y = e^{\alpha x} v_y \quad \text{on } y = 1, \tag{14}$$
$$\text{and} \quad e^{-\alpha x} u + e^{\alpha x} v = 2(ch^2 \alpha x - ch^2 \alpha)/(\alpha sh2\alpha), \quad \text{on } y = 1. \tag{15}$$

Note that now the domain is $[-1,1] \times [0,1]$ due to symmetry about x-axis.

As shown by Grinberg [1] the solutions of (10) and (11) satisfying all boundary conditions except the last one can be expressed as

$$u(x,y) = \frac{1}{2\pi} \int_{-1}^{1} G(x,y,t) F(t) e^{\alpha t} dt, \tag{16}$$
$$v(x,y) = \frac{1}{2\pi} \int_{-1}^{1} G(x,y,t) F(t) e^{-\alpha t} dt, \tag{17}$$
$$\text{where} \quad F(x) = e^{-\alpha x} u_y = e^{\alpha x} v_y, \quad -1 \leqslant x \leqslant 1, \tag{18}$$
$$G(x,y,t) = \sum_m \sum_n (-1)^n K_0(\alpha r_{mn}), \tag{19}$$
$$r_{mn}^2 = [2n + (-1)^n x - t]^2 + [m - 1/2 + (-1)^m (y - 1/2)]^2, \tag{20}$$
$$\text{and} \quad K_0 = \text{modified Bessel function}. \tag{21}$$

The summations in (19) are over all integral values of m and n. The function $G(x,y,t)$ is the Green's function for the problem. Finally, satisfying the last boundary condition we get the integral equation

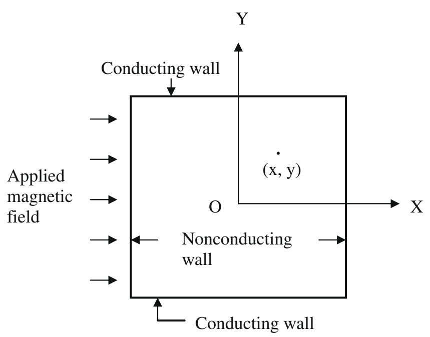

Fig. 1. Flow out of paper.