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Global exponential stability for shunting inhibitory CNNs with delays ☆

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ABSTRACT

In this paper, the global exponential stability for shunting inhibitory cellular neural networks (SICNNs) with delays is studied by constructing suitable Lyapunov functionals and applying some critical analysis techniques. The sufficient conditions guaranteeing the network's global exponential stability are obtained. Our results impose less restrictive conditions than those in the references. Moreover an example is given to illustrate the feasibility of the conditions in our results.

1. Introduction

Recently, cellular neural networks (CNNs) have shown great potential as information-processing systems, and many researchers have paid much attention to the research on the theory and application of the CNNs (see [1–7]). Some sufficient conditions are given to ensure the existence and stability of the equilibrium point for the CNNs (see [1–5]), and some good research results are given for some other neural networks (see [8–12]). The shunting inhibitory cellular neutral networks (SICNNs) are a new class of CNNs, they were introduced by Bouzerdoum and Pinter in [13]. Now SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, and image processing. Consider a two-dimensional grid of processing cells, let c_{ij} denote the cell at the (i,j) position of the lattice, the r_neighborhood $N_r(i,j)$ of c_{ij} is

$$N_r(i,j) = \{c_{kl} : \max(|k-i|, |l-j|) \leqslant r,\ 1 \leqslant k,l \leqslant n\}.$$

In SICNNs, neighboring cells exert mutual inhibitory interactions of the shunting type. The dynamics of a cell c_{ij} are described by the following nonlinear ordinary differential equation:

$$\frac{dx_{ij}(t)}{dt} = -a_{ij}x_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} c_{kl}f(x_{kl}(t))x_{ij}(t) + L_{ij}(t), \quad 1 \leqslant i,j \leqslant n, \tag{1}$$

where x_{ij} is the activity of the cell c_{ij}, $L_{ij}(t)$ is the external input to c_{ij}, the constant $a_{ij} > 0$ represents the passive decay rate of the cell activity, $c_{kl} \geqslant 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell c_{ij}, the activation function $f(x_{kl})$ is a positive continuous function representing the output or firing rate of the cell c_{kl}.

When we consider the influence of the delay, the system (1) can be modified to the following form:

$$\frac{dx_{ij}(t)}{dt} = -a_{ij}x_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} c_{kl}f(x_{kl}(t-\tau))x_{ij}(t) + L_{ij}(t), \quad 1 \leqslant i,j \leqslant n. \tag{2}$$

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In this paper, we consider the special case of the system (2) as $L_{ij}(t) = L_{ij}$, i.e.,

$$\frac{dx_{ij}(t)}{dt} = -a_{ij}x_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} c_{kl}f(x_{kl}(t-\tau))x_{ij}(t) + L_{ij}, \quad 1 \leqslant i,j \leqslant n, \tag{3}$$

where L_{ij} $(i,j \in [1,n])$ is constant.

For SICNNs, the existence and attractivity of almost periodic solution were considered in [14–20]. Up to now, to the best of our knowledge, no author studies the exponential stability of the equilibrium of SICNNs.

The purpose of this paper is to give out some sufficient conditions for global exponential stability of system (3). We use the theory of Brouwer degree, rather than the fixed point method used in many previous references, to prove the existence of the equilibrium. By constructing suitable Lyapunov functions, we prove the global exponential stability of the equilibrium. Our results impose less restrictive conditions than those in [14–20], so our results are beneficial in improving the study of stability of SICNNs.

2. Global exponential stability conditions for shunting inhibitory CNNs with delays

Lemma 2.1. *Assume x^* is the equilibrium of system (3). Then*

$$\left|x_{ij}^*\right| \leqslant k = \max_{ij}\left\{\frac{|L_{ij}|}{a_{ij}}\right\}.$$

Proof. x^* satisfies

$$-a_{ij}x_{ij}^* - \sum_{c_{kl}\in N_r(i,j)} c_{kl}f(x_{kl}^*)x_{ij}^* + L_{ij} = 0.$$

Therefore,

$$\left|x_{ij}^*\right| = \frac{|L_{ij}|}{a_{ij} + \sum_{c_{kl}\in N_r(i,j)} f(x_{kl}^*)} \leqslant \frac{|L_{ij}|}{a_{ij}} \leqslant k = \max_{ij}\left\{\frac{|L_{ij}|}{a_{ij}}\right\}.$$

This completes the proof. □

Theorem 2.2. *For system (3), assume that the following condition is satisfied*

$$(H_1) \quad a_{ij} > M_f \sum_{c_{kl}\in N_r(i,j)} c_{kl}, \quad \forall 1 \leqslant i,j \leqslant n.$$

Then there exists an equilibrium for system (3), where $M_f = \sup_{x\in R}|f(x)|$.

Proof. Define a mapping:

$$F(x) = (F_{11}(x), F_{12}(x), \ldots, F_{1n}(x), F_{21}(x), \ldots, F_{2n}(x), \ldots, F_{nn}(x))^T, \tag{4}$$

where

$$x = (x_{11}, x_{12}, \ldots, x_{1n}, x_{21}, \ldots, x_{2n}, \ldots, x_{nn})^T,$$
$$F_{ij}(x) = a_{ij}x_{ij} + \sum_{c_{kl}\in N_r(i,j)} c_{kl}f(x_{kl}(t-\tau))x_{ij} - L_{ij}, \quad 1 \leqslant i,j \leqslant n.$$

If $F(x) = 0$ has a solution, then system (3) will have an equilibrium. Set another mapping:

$$G(x,\lambda) = \lambda F(x) + (1-\lambda)x, \tag{5}$$

where $\lambda \in [0,1]$, and

$$G(x,\lambda) = (G_{11}(x,\lambda), G_{12}(x,\lambda), \cdots, G_{1n}(x,\lambda), G_{21}(x,\lambda), \ldots, G_{2n}(x,\lambda), \ldots, G_{nn}(x,\lambda))^T.$$

From (4) and (5), it is easy to get

$$|G_{ij}(x,\lambda)| = \left|\lambda a_{ij}x_{ij} + \lambda \sum_{c_{kl}\in N_r(i,j)} c_{kl}f(x_{kl}(t-\tau))x_{ij} - \lambda L_{ij} + (1-\lambda)x_{ij}\right|$$
$$\geqslant (\lambda a_{ij} + (1-\lambda))|x_{ij}| - \lambda \sum_{c_{kl}\in N_r(i,j)} c_{kl}|f(x_{kl}(t-\tau))||x_{ij}| - \lambda|L_{ij}| \geqslant \lambda\left(a_{ij} - M_f \sum_{c_{kl}\in N_r(i,j)} c_{kl}\right)|x_{ij}| - \lambda|L_{ij}|.$$

Hence, $\forall j \in [1,n]$

$$\sum_{i=1}^{n} |G_{ij}(x,\lambda)| \geqslant \lambda \sum_{i=1}^{n}\left(a_{ij} - M_f \sum_{c_{kl}\in N_r(i,j)} c_{kl}\right)|x_{ij}| - \lambda \sum_{i=1}^{n} |L_{ij}|.$$