Applied Mathematics and Computation 214 (2009) 310–317

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Nonnegative doubly periodic solutions for nonlinear telegraph system with twin-parameters

Fanglei Wang [a,*], Wanjun Li [b], Yukun An [a]

[a] Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China
[b] Department of Mathematics, Longdong University Qingyang, Gansu 745100, PR China

ARTICLE INFO

Keywords:
Telegraph system
Doubly periodic solution
Cone
Fixed-point theorem

ABSTRACT

In this paper, by using Krasnosel'skii fixed-point theorem and under suitable conditions, we present the existence and multiplicity of nonnegative doubly periodic solutions for the following system:

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_{11}(t,x)u + a_{12}(t,x)v = b_1(t,x)f(t,x,u,v), \\ v_{tt} - v_{xx} + c_2 v_t + a_{21}(t,x)u + a_{22}(t,x)v = b_2(t,x)g(t,x,u,v), \end{cases}$$

where $c_i > 0$ is a constant, $a_{11}(t,x), a_{22}(t,x), b_1(t,x), b_2(t,x) \in C(R^2, R^+), a_{12}(t,x), a_{21}(t,x) \in C(R^2, R^-), f(t,x,u,v), g(t,x,u,v) \in C(R^2 \times R^+ \times R^+, R^+)$, and a_{ij}, b_i, f, g are 2π-periodic in t and x. We derive two explicit intervals of $b_1(t,x)$ and $b_2(t,x)$ such that for any $b_1(t,x)$ and $b_2(t,x)$ in the two intervals respectively, the existence of at least one solution for the system is guaranteed, and the existence of at least two solutions for $b_1(t,x)$ and $b_2(t,x)$ in appropriate intervals is also discussed.

1. Introduction

In this paper we are concerned with the existence and multiplicity of nonnegative solutions for the nonlinear telegraph system

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_{11}(t,x)u + a_{12}(t,x)v = b_1(t,x)f(t,x,u,v) \quad (t,x) \in R^2, \\ v_{tt} - v_{xx} + c_2 v_t + a_{21}(t,x)u + a_{22}(t,x)v = b_2(t,x)g(t,x,u,v) \quad (t,x) \in R^2, \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x) \quad (t,x) \in R^2, \\ v(t+2\pi,x) = v(t,x+2\pi) = v(t,x) \quad (t,x) \in R^2. \end{cases} \tag{1}$$

The existence of a doubly periodic solution for a single telegraph equation is studied by many authors when the nonlinearity is bounded or linear growth, see [1–4] and the references therein. The first maximum principle for linear telegraph equations was built by Ortega and Robles-Perez in [4]. They proved that the maximum principle for the doubly 2π-periodic solutions of the linear telegraph equation

$$u_{tt} - u_{xx} + cu_t + \lambda u = h(t,x) \quad (t,x) \in R^2$$

holds if and only if $\lambda \in (0, \nu(c)]$, where $\nu(c) \in \left(\frac{c^2}{4}, \frac{c^2}{4} + \frac{1}{4}\right)$ is a constant which cannot be concretely determined. This maximum principle on the torus $\top^2$ (here $\top = R/2\pi Z$ denotes the unit circle) was used in [4] to develop a method of upper and lower solutions for the doubly periodic solutions of the nonlinear telegraph equation

* Corresponding author.
E-mail addresses: wang-fanglei@hotmail.com (F. Wang), anyksd@hotmail.com (Y. An).

0096-3003/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2009.03.069

$$u_{tt} - u_{xx} + cu_t + v(c)u = F(t,x,u) \quad (t,x) \in R^2,$$

when the function $u \mapsto F(t,x,u) + v(c)u$ is monotonically nondecreasing. Afterwards in [5], Mawhin et al. built a maximum principle for the solutions $u(t,x)$ of the telegraph equation which are 2π-periodic with respect to x and bounded on R with respect to t when $\lambda \in \left(0, \frac{c^2}{4}\right]$. And a similar method of upper and lower solutions was developed when the function $u \mapsto F(t,x,u) + v(c)u$ is monotonically nondecreasing. Lately, these authors in [6] have extended their results in [5] to the telegraph equations in space dimensions two or three. In addition, another maximum principle for the telegraph equation can be found in [7], which was built by Li. Recently, by using fixed-point theorem in cones in [8], Li obtained the existence results of positive doubly periodic solutions for the nonlinear equation

$$u_{tt} - u_{xx} + cu_t + a(t,x)u = b(t,x)f(t,x,u), \tag{2}$$

where $c > 0$ is a constant, $a(t,x), b(t,x) \in C(R^2, R^+), f(t,x,u) \in C(R^2 \times R^+, R^+)$, and $a(t,x), b(t,x), f(t,x,u)$ are 2π-periodic in t and x, $0 \leqslant a(t,x) \leqslant c^2/4$ and f is either superlinear or sublinear on u on the base of maximum principle in [4].

On the other hand, there are many papers connected with the existence and multiplicity of t-periodic solutions for the nonlinear wave systems and telegraph-wave coupled systems, such as [9–11]. And also many authors deal with second-order ordinary differential systems and second-order elliptic systems, see [12–16].

Inspired by those papers, here our interest is to establish some simple criteria for the existence of single and multiple solutions for the nonlinear telegraph systems (1) in explicit intervals of $b_1(t,x)$ and $b_2(t,x)$ by using Krasnoselskii fixed-point theorem.

The paper is organized as follows: In Section 2, we make some preliminaries; in Section 3, we discuss the existence of single positive solution for the systems (1); in Section 4, we study the existence conditions of at least two positive solutions for the systems (1).

2. Preliminaries

Let $\top^2$ be the torus defined as

$$\top^2 = (R/2\pi Z) \times (R/2\pi Z).$$

Doubly 2π-periodic functions will be identified to be functions defined on $\top^2$. We use the notations

$$L^p(\top^2), C(\top^2), C^\alpha(\top^2), D(\top^2) = C^\infty(\top^2), \ldots$$

to denote the spaces of doubly periodic functions with the indicated degree of regularity. The space $D'(\top^2)$ denotes the space of distributions on $\top^2$.

By a doubly periodic solution of (1) we mean that a $(u,v) \in L^1(\top^2) \times L^1(\top^2)$ satisfies (1) in the distribution sense, i.e.

$$\begin{cases} \int_{\top_2} u(\varphi_{tt} - \varphi_{xx} - c_1\varphi_t + a_{11}\varphi) + \int_{\top_2} a_{12} v\varphi = \int_{\top^2} b_1 f\varphi, \\ \int_{\top_2} v(\varphi_{tt} - \varphi_{xx} - c_2\varphi_t + a_{22}\varphi) + \int_{\top_2} a_{21} u\varphi = \int_{\top^2} b_2 g\varphi \quad \forall \varphi \in D(\top^2). \end{cases}$$

First, we consider the linear equation

$$u_{tt} - u_{xx} + c_i u_t - \lambda_i u = h(t,x) \quad \text{in } D'(\top^2), \tag{3}$$

where $c_i > 0, \lambda_i \in R, h(t,x) \in L^1(\top^2)$ $(i = 1,2)$.

Let $£_{\lambda_i}$ be the differential operator

$$£_{\lambda_i} = u_{tt} - u_{xx} + c_i u_t - \lambda_i u$$

acting on functions on $\top^2$. From [4,8], it is easy to know that if $\lambda_i < 0$, $£_{\lambda_i}$ has the resolvent R_{λ_i}

$$R_{\lambda_i} : L^1(\top^2) \to C(\top^2), \quad h \mapsto u_i,$$

where u_i is the unique solution of Eq. (3), and the restriction of R_{λ_i} on $L^p(\top^2)$ $(1 < p < \infty)$ or $C(\top^2)$ is compact. In particular, $R_{\lambda_i} : C(\top^2) \to C(\top^2)$ is a completely continuous operator.

For $\lambda_i = -c_i^2/4$, the Green function of the differential operator $£_{\lambda_i}$ is explicitly expressed, which has been obtained in [4]. We denote it by $G_i(t,x)$. By Lemma 5.1 in [4], $G_i(t,x) \in L^\infty$, is doubly 2π-periodic. So the unique solution of Eq. (3) can be represented by convolution product

$$u_i(t,x) = (R_{\lambda_i} h)(t,x) = \int_{\top^2} G_i(t-s, x-y) h(s,y)\, ds\, dy. \tag{4}$$

The expression of $G_i(t,x)$ will be given in the following.

Let $D_i = R^2 \setminus \zeta_i$, where ζ_i is the family of lines

$$x \pm t = 2k\pi, \quad k \in Z.$$