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Nonlinear dynamic research on EEG signals in HAI experiment

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Abstract

According to the phase space reconstruction techniques from one-dimensional time series and the quantitative criteria of chaotic system, analyses and computations are conducted on Electroencephalogram (EEG) signals of piglets resulted from HAI (hypoxic–asphyxic injury) experiments. Based on the comparative studies of the phase graph, the power spectra, the correlation dimension, and the Lyapunov exponent of EEG signals of piglets, under normal and injury conditions, the following conclusions are shown: (1) the analyses of phase graph, power spectra, correlation dimension and Lyapunov exponent of EEG signals manifest the whole dynamic characteristics of cerebrum, and they may be used as new quantitative methods for early diagnosis of brain injuries. (2) Under normal physiological conditions, EEG signals are chaotic; while under pathologic conditions, EEG signals approach regularity.

Keywords: Chaos; EEG signal; HAI experiment; Phase graph; Power spectra; Correlation dimension; Lyapunov exponent

1. Introduction

Electroencephalogram (EEG) signal is the spontaneous bioelectric activity of the cerebral cortex. It has been widely used in clinical studies and nerve electrophysiology researches due to its abundant information about estates and changes of neural systems. In recent years, with the developments of nonlinear dynamics, more and more evidence shows that cerebrum is a nonlinear dynamical system and EEG signals can be regarded as its outputs [1,2]. Therefore, in order to get a break-through in cerebrum knowledge [3], people try to analyze EEG signals based on nonlinear dynamic methods. In 1985, Babloyantz and his colleagues suggested for the first time that phase II and phase IV in sleep EEG signals of human beings are chaotic [4]. Afterwards, it has been reported in many studies that EEG originates from chaotic systems [5–15]. In 2002, Ferri et al. investigated the nonlinear dynamics of sleep EEG by applying nonlinear cross prediction [13]. In 2004, Keshavan et al. researched the decreased nonlinear complexity of sleep EEG in first episode schizophrenia by analyzing the symbolic dynamics and the largest Lyapunov exponent of the signals [14]. In 2005, Tong et al. studied the complexity of human EEG in visual processing by computing the fractal dimensions [15].

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However, Glass thought it not creditable that EEG was claimed to be chaotic only according to the computation results of some nonlinear dynamical methods (such as fractal dimension and Lyapunov exponent) [16,17]. We consider that it is not persuasive enough to illustrate that EEG signals of a healthy body is more chaotic than that of a morbid one simply with the actual data collected from different human bodies [7,12–15]. Resulted from individual differences or pathologies, it is difficult to determine whether this tendency is to order or to chaos. Therefore, in this paper, experiments are conducted on the same body under different conditions, and the data under normal and pathological situations are compared. Glass proposed a method that by changing a certain important parameter controlling the body, physiological signals can be transformed from periodic dynamics to chaotic dynamics. If this process can be illustrated with experimental data, it then can be safely concluded that the observed dynamical characteristics are chaotic [16,17]. In this paper, we adopted chaos theory to the studies on EEG signals of piglets in HAI experiments and we found that EEG signals under normal physiological conditions possess certain inherent changeability which coincides with chaotic states, and the loss of the changeability is the foreboding of cerebrum injury.

2. Theory and method

Chaotic systems can be described by strange attractors in the phase space [3,18]. In order to construct the phase space, we adopt the phase space reconstruction technology, which was put forward by Packard [19] and was made reliable mathematical basis by Takens [20]. Its principal is: reconstruct m-dimensional phase space with time series $\{x_n | n = 1, 2, \dots, N\}$, then we can get a group of phase space vectors:

$$\vec{X}_i = \{x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}\}; \quad i = 1, 2, \dots, M; \; \vec{X}_i \in R^m, \tag{1}$$

where τ is the time-delay; $m \geqslant 2d + 1$, and d is the number of the system independent variables. M is less than N and they have the same order of magnitude. The phase space reconstruction is crucial to the analysis of phase graph, correlation dimension and Lyapunov exponent.

2.1. Power spectra

In this paper, the estimated values of self-power spectra about EEG signals are calculated using the autoregressive (AR) parametric model [21]. That is, the EEG signal $x(t)$ is denoted by the discrete time series x_n:

$$x_n = -\sum_{k=1}^{p} a_k x_{n-k} + w_n, \tag{2}$$

where p is the order of the AR process; a_k $(k = 1, 2, \dots, p)$ is parameter; and w_n is the unpredictable part of x_n, *i.e.*, the residue error. If the AR process can well match the EEG time series, w_n will be white noise. From Eq. (2), we can get the estimated value of power spectra,

$$P_x(\omega) = \frac{\sigma_\omega^2}{|A(\mathrm{e}^{\mathrm{j}\omega})|^2} = \frac{\sigma_\omega^2}{\left|1 + \sum_{k=1}^{p} a_k \mathrm{e}^{-\mathrm{j}\omega k}\right|^2}, \tag{3}$$

where σ_ω^2 is the mean-square error of the residue error. According to Eqs. (2) and (3), we know that it is essential to estimate the AR parameters a_k $(k = 1, 2, \dots, p)$ from EEG time series in order to get the estimated value of power spectra. In general, we adopt Yule–Walker equation and Levinson–Durbin arithmetic to estimate the AR parameters.

In addition, the value of p (the order of the AR process) is very important because slippery estimated value of power spectra will appear when the order p is too low, while false spike and general statistical instability will appear if the order p is too high. Today, almost all estimates of the model order are based on the computation of forecast error power. In this paper, Akaike information criterion (AIC) [22] is employed to estimate the order,

$$\mathrm{AIC}(p) = N \ln \widehat{\rho}_p + 2p, \tag{4}$$

where N is the maximum number of analysis samples and $\widehat{\rho}_p$ is the estimated variance of the white noise when the order is p.