Applied Mathematics and Computation 212 (2009) 185–193

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity

Ching-Yang Cheng

Department of Mechanical Engineering, Southern Taiwan University, Yungkang 71005, Taiwan

ARTICLE INFO

Keywords:
Nonsimilar solutions
Coordinate transformation
Cubic spline collocation method
Double diffusion
Truncated cone
Porous medium
Variable viscosity

ABSTRACT

This work presents a boundary layer analysis about variable viscosity effects on the double-diffusive convection near a vertical truncated cone in a fluid-saturated porous medium with constant wall temperature and concentration. The viscosity of the fluid is assumed to be an inverse linear function of the temperature. A boundary layer analysis is employed to derive the nondimensional nonsimilar governing equations, and the transformed boundary layer governing equations are solved by the cubic spline collocation method to yield computationally efficient numerical solutions. The obtained results are found to be in good agreement with previous papers on special cases of the problem. Results for local Nusselt and Sherwood numbers are presented as functions of viscosity-variation parameter, buoyancy ratio, and Lewis number. For a porous medium saturated with a Newtonian fluid with viscosity proportional to an inverse linear function of temperature, higher value of viscosity-variation parameter leads to the decrease of the viscosity in fluid flow, thus increasing the fluid velocity as well as the local Nusselt number and the local Sherwood number.

1. Introduction

Double-diffusive convection by buoyancy due to temperature and concentration gradients in a fluid-saturated porous medium may be met in a lot of geophysical, geothermal and industrial applications, such as the migration of moisture through air contained in fibrous insulations and the underground spreading of chemical contaminants through water-saturated soil.

Bejan and Khair [1] examined the free convection boundary layer flow driven by both temperature and concentration gradients. Cheng et al. [2] studied the natural convection of a Darcian fluid about a truncated cone. Lai and Kulacki [3] studied the natural convection boundary layer along a vertical surface with constant heat and mass flux and the effect of wall injection. Yih [4] studied the heat and mass transfer driven by natural convection from a truncated cone embedded in a porous medium with variable wall temperature and concentration or with variable heat and mass flux. Cheng [5] studied the problem of free convection heat and mass transfer near a wavy cone in a porous medium.

The fluid viscosity may have a significant change with temperature in some cases, and the researcher had better take into account the variation of viscosity to obtain a better estimation of the flow and heat and mass transfer behavior. Gray et al. [6] studied the effect of significant viscosity-variation on heat transfer in water-saturated porous media. Lings and Dybbs [7] examined the forced convection over a flat plate in porous media with variable viscosity proportional to an inverse linear function of temperature. Kafoussius and Williams [8] studied the effect of temperature dependent viscosity on the free convection laminar boundary layer flow along a vertical isothermal plate. Kafoussius and Rees [9] examined the effect of

E-mail address: cycheng@mail.stu.edu.tw

0096-3003/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2009.02.012

Nomenclature

A	half angle of the cone
C	concentration
D	mass diffusivity
E	viscosity-variation parameter
f	dimensionless stream function
g	gravitational acceleration
h	heat transfer coefficient
h_m	mass transfer coefficient
K	permeability of the porous medium
k	thermal conductivity
Le	Lewis number
N	buoyancy ratio
Nu	Nusselt number
Ra	Darcy-modified Rayleigh number
r	local radius of the truncated cone
Sh	Sherwood number
T	temperature
u, v	velocity components
x	streamwise coordinates
$\bar{x}$	modified streamwise coordinates
x_0	distance of the leading edge of the truncated cone measured from the origin
y	transverse coordinates

Greek symbols

α	thermal diffusivity
β_c	coefficient of concentration expansion
β_t	coefficient of thermal expansion
η	transverse coordinates
θ	dimensionless temperature
μ	viscosity
ξ	dimensionless streamwise coordinate
ρ	density
ϕ	dimensionless concentration
ψ	stream function
ω	viscosity-variation constant

Subscripts

w	condition at wall
∞	condition at infinity

temperature dependent viscosity on the mixed convection laminar boundary layer flow along a vertical isothermal plate. Hassanien et al. [10] studied the variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media for the entire regime. Al-Harbi [11] performed a numerical study of natural convection heat transfer with variable viscosity and thermal radiation from a cone and wedge in porous media. Molla et al. [12] studied the natural convection flow from an isothermal circular cylinder with temperature dependent viscosity. Cheng [13] examined the effect of temperature dependent viscosity on the natural convection heat transfer from a horizontal isothermal cylinder of elliptic cross section. Jayanthi and Kumari [14,15] studied the effect of variable viscosity on non-Darcy free or mixed convection flow on a vertical surface in a porous medium saturated with a Newtonian fluid or a non-Newtonian fluid. Chin et al. [16] studied the effect of variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium.

In this paper, we want to extend the works of Yih [4] and Lings and Dybbs [7] to study the heat and mass transfer driven by natural convection along the surface of a frustum of a cone in a porous medium saturated with a Newtonian fluid with viscosity proportional to an inverse linear function of temperature. The transformed nonsimilar boundary layer governing equations are solved by the cubic spline collocation method [17,18]. The effects of the viscosity-variation parameter, the Lewis number, and the buoyancy ratio on the local Nusselt number and the Sherwood number for natural convection heat and mass transfer near a vertical frustum of a cone in a fluid-saturated porous medium are carefully examined.