Applied Mathematics and Computation 207 (2009) 333–339

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Geodesic-pancyclicity and fault-tolerant panconnectivity of augmented cubes

Hung–Chang Chan [a], Jou–Ming Chang [b,*], Yue–Li Wang [c], Shi–Jinn Horng [d,e]

[a] Department of Computer Science and Information Engineering, Yuanpei University, Hsinchu, Taiwan, ROC
[b] Department of Information Management, National Taipei College of Business, No. 321, Sec. 1, Ji-Nan Rd., Taipei 10051, Taiwan, ROC
[c] Department of Information Management, National Taiwan University of Science and Technology, Taipei, Taiwan, ROC
[d] Department of Electronic Engineering, National United University, Miao-Li, Taiwan, ROC
[e] Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, ROC

ARTICLE INFO

Keywords:
Pancyclicity
Geodesic-pancyclicity
Panconnectivity
Augmented cubes
Interconnection networks

ABSTRACT

Choudum and Sunitha [S.A. Choudum, V. Sunitha, Augmented cubes, Networks 40 (2002) 71–84] proposed the class of augmented cubes as a variation of hypercubes and showed that augmented cubes possess several embedding properties that the hypercubes and other variations do not possess. Recently, Hsu et al. [H.-C. Hsu, P.-L. Lai, C.-H. Tsai, Geodesic-pancyclicity and balanced pancyclicity of augmented cubes, Information Processing Letters 101 (2007) 227–232] showed that the n-dimensional augmented cube AQ_n, $n \geqslant 2$, is weakly geodesic-pancyclic, i.e., for each pair of vertices $u, v \in AQ_n$ and for each integer ℓ satisfying $\max\{2d(u,v), 3\} \leqslant \ell \leqslant 2^n$ where $d(u,v)$ denotes the distance between u and v in AQ_n, there is a cycle of length ℓ that contains a u-v geodesic. In this paper, we obtain a stronger result by proving that AQ_n, $n \geqslant 2$, is indeed geodesic-pancyclic, i.e., for each pair of vertices $u, v \in AQ_n$ and for each integer ℓ satisfying $\max\{2d(u,v), 3\} \leqslant \ell \leqslant 2^n$, every u-v geodesic lies on a cycle of length ℓ. To achieve the result, we first show that $AQ_n - f$, $n \geqslant 3$, remains panconnected (and thus is also edge-pancyclic) if $f \in AQ_n$ is any faulty vertex. The result of fault-tolerant panconnectivity is the best possible in the sense that the number of faulty vertices in AQ_n, $n \geqslant 3$, cannot be increased.

1. Introduction

An interconnection network is usually modeled as an undirected simple graph $G = (V,E)$, where the vertex set $V = V(G)$ denotes the set of processing elements and the edge set $E = E(G)$ denotes the set of communication channels, respectively. For interconnection networks, nicely topological properties enable them to support efficient and robust communication algorithms. Especially, properties related to cycle embedding have attracted a burst of studies in the literature (see, for example, [1,7–9,12,19]) because networks with cycle topology are suitable for designing simple algorithms with low communication costs. Moreover, cycle embeddings are also concerned extensively in many diverse interconnection networks with faulty elements [4,13–17,20,21]. For more information about the interconnection networks, the reader can refer to [10,18].

Let G be a graph and $u, v \in V(G)$ be any two vertices. A path (respectively, shortest path) connecting u and v is called a *u-v path* (respectively, *u-v geodesic*). The *distance* between u and v, denoted by $d_G(u,v)$, is the number of edges in a u-v geodesic. A path (respectively, cycle) that contains every vertex of a graph exactly once is called a *hamiltonian path*

* Corresponding author.
E-mail address: spade@mail.ntcb.edu.tw (J.-M. Chang).

0096-3003/$ - see front matter © 2008 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2008.10.061

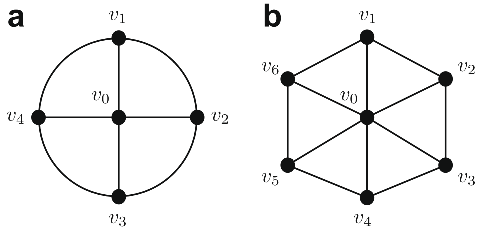

Fig. 1. (a) A weakly geodesic-pancyclic graph that is not geodesic-pancyclic; (b) a panconnected graph that is not weakly geodesic-pancyclic.

(respectively, *hamiltonian cycle*). A graph G is *traceable* (respectively, *hamiltonian*) if it possesses a hamiltonian path (respectively, a hamiltonian cycle). Recently, properties stronger than hamiltonicity have been widely considered in many interconnection networks. A graph G is *hamiltonian-connected* if every two vertices of G are connected by a hamiltonian path. A graph G is *pancyclic* if it contains a cycle of length ℓ (i.e., an ℓ-cycle) for each ℓ with $3 \leqslant \ell \leqslant |V|$. In particular, G is called *vertex-pancyclic* (respectively, *edge-pancyclic*) if every vertex (respectively, edge) of G belongs to an ℓ-cycle for each ℓ with $3 \leqslant \ell \leqslant |V|$. A graph G is *panconnected* if, for any two distinct vertices u, $v \in V$ and for each integer ℓ with $d_G(u, v) \leqslant \ell \leqslant |V| - 1$, there is a u-v path of length ℓ in G. The existence of pancyclicity and/or panconnectivity in a network is even more important because it implies that the network can embed cycles and/or paths with arbitrary length [4,5].

An enhancement of cycle embedding using geodesic as a part of the cycle was recently suggested by [3]. A pair of vertices $\langle u, v \rangle$ in a graph G is said to be *geodesic-pancyclic* (respectively, *weakly geodesic-pancyclic*) if for each integer ℓ satisfying $\max\{2d_G(u, v), 3\} \leqslant \ell \leqslant |V|$, every u-v geodesic lies on an ℓ-cycle (respectively, there is an ℓ-cycle that contains a u-v geodesic). A graph G is called *geodesic-pancyclic* (respectively, *weakly geodesic-pancyclic*) if, for every two vertices u, $v \in V(G)$, $\langle u, v \rangle$ is geodesic-pancyclic (respectively, *weakly geodesic-pancyclic*). Obviously, every geodesic-pancyclic graph is weakly geodesic-pancyclic, and the converse in not true. For instance, the graph shown in Fig. 1a is weakly geodesic-pancyclic but it is not geodesic-pancyclic, where a geodesic $v_1v_0v_3$ does not lie on any 5-cycle. In [3], Chan et al. studied sufficient conditions of geodesic-pancyclic graphs and showed that most of the known sufficient conditions of panconnected graphs can be applied to geodesic-pancyclic graphs. Note that if a graph G is weakly geodesic-pancyclic or panconnected then clearly it is edge-pancyclic. However, both the classes of weakly geodesic-pancyclic graphs and panconnected graphs are not identical. A graph G which is panconnected does not have to be weakly geodesic-pancyclic. For instance, the graph shown in Fig. 1b is panconnected, while there does not exist a 4-cycle containing the unique v_1–v_4 geodesic $v_1v_0v_4$. It remains an open question whether every geodesic-pancyclic graph is panconnected [3].

In this paper, we study the geodesic-pancyclicity and fault-tolerant panconnectivity for a particular family of interconnection networks called augmented cubes. The augmented cube AQ_n, proposed by Choudum and Sunitha [6], is a variation of hypercubes. Let F be a set of faulty elements in a graph G and G–F denotes the residual graph of G by removing the faulty elements. Hsu et al. [11] proved that, for $n \geqslant 4$, $AQ_n - F$ is hamiltonian if $|F| \leqslant 2n - 3$ and is hamiltonian-connected if $|F| \leqslant 2n - 4$, where F is any subset of $V(AQ_n) \cup E(AQ_n)$. Ma et al. [14] showed that AQ_n is panconnected for $n \geqslant 1$ and $AQ_n - F$ is pancyclic if $|F| \leqslant 2n - 3$ for every $F \subset E(AQ_n)$ and $n \geqslant 2$. Wang et al. [17] showed that $AQ_n - F$ remains pancyclic provided $|F| \leqslant 2n - 3$ for every $F \subset V(AQ_n) \cup E(AQ_n)$ and $n \geqslant 4$. In addition, Hsu et al. [12] also proved that AQ_n is weakly geodesic-pancyclic. Note that the term 'weakly geodesic-pancyclicity' is named as 'geodesic-pancyclicity' in that paper. In this paper, we obtain a more strong result by proving that AQ_n is indeed geodesic-pancyclic for $n \geqslant 2$. To achieve this property, a preliminary result shows that $AQ_n - f$, $n \geqslant 3$, is still panconnected if f is any faulty vertex of AQ_n.

The rest of this paper is organized as follows. Section 2 gives the definition of AQ_n and discuss some properties of AQ_n. Section 3 presents the fault-tolerant panconnectivity of AQ_n. Section 4 proves that AQ_n is geodesic-pancyclic. The last section contains our concluding remarks.

2. Structural properties of AQ_n

The n-dimension augmented cube AQ_n $(n \geqslant 1)$ has 2^n vertices, each vertex is labeled by an n-bit binary string, and can be defined recursively as follows. AQ_1 is a complete graph K_2 with the vertex set {0,1}. For $n \geqslant 2$, AQ_n is constructed by taking two copies of AQ_{n-1}, denoted by AQ^0_{n-1} and AQ^1_{n-1} with $V(AQ^k_{n-1}) = \{ku_{n-1}u_{n-2}\dots u_1 : u_i \in \{0, 1\}$ and $1 \leqslant i \leqslant n - 1\}$ for $k \in \{0, 1\}$, and adding 2^n edges between AQ^0_{n-1} and AQ^1_{n-1} by the following rule. A vertex $u = 0u_{n-1}u_{n-2}\dots u_1$ of AQ^0_{n-1} is joined to a vertex $v = 1v_{n-1}v_{n-2}\dots v_1$ of AQ^1_{n-1} if and only if either

(i) $u_i = v_i$ for all $1 \leqslant i \leqslant n - 1$ (in this case, uv is called a *hypercube edge*), or
(ii) $u_i = \bar{v}_i$ for all $1 \leqslant i \leqslant n - 1$ (in this case, uv is called a *complement edge*).