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Application of the $\left(\frac{G'}{G}\right)$-expansion to travelling wave solutions of the Broer–Kaup and the approximate long water wave equations

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ABSTRACT

By using the $\left(\frac{G'}{G}\right)$-expansion proposed recently the travelling wave solutions involving parameters of the Broer–Kaup equations and the approximate long water wave equations are found out. The travelling wave solutions are expressed by three types of functions which are the hyperbolic functions, the trigonometric functions and the rational functions. When the parameters are taken as special values the solitary wave solutions are obtained.

1. Introduction

In recent years, searching for explicit solutions of nonlinear evolution equations by using various methods has become the main goal for many authors, and many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the tanh-function expansion and its various extension [1,2], the Jacobi elliptic function expansion [3,4], the *F*-expansion [5–8], the sub-ODE method [9–12], the homogeneous balance method [13–15], the sine–cosine method [16,17] and the Exp-function method [18], and so on but up to now a unified method that can be used to deal with all types of nonlinear evolution equations has not been discovered.

In the present paper we consider two systems of nonlinear evolution equations: one is the Broer–Kaup equations [19] in the form

$$u_t + uu_x + v_x = 0, \tag{1.1}$$
$$v_t + u_x + (uv)_x + u_{xxx} = 0 \tag{1.2}$$

and another is the approximate long water wave equations [20–22] in the form

$$u_t - uu_x - v_x + \alpha u_{xx} = 0, \tag{1.3}$$
$$v_t - (uv)_x - \alpha v_{xx} = 0. \tag{1.4}$$

The main goal of this paper is that using the $\left(\frac{G'}{G}\right)$-expansion [23] proposed recently, we will find out more types of travelling wave solutions involving parameters of Eqs. (1.1), (1.2) and (1.3), (1.4). The key ideas of the $\left(\frac{G'}{G}\right)$-expansion method are that the travelling wave solutions of nonlinear evolution equations can be expressed by polynomials in $\left(\frac{G'}{G}\right)$, where $G = G(\xi)$ satisfies a second order linear differential equation (LODE for short), $G' = \frac{dG}{d\xi}$, $\xi = x - Vt$, the degree of the polynomials can be determined by considering the homogeneous balance between the highest order partial derivatives and nonlinear terms appearing in nonlinear evolution equations considered, the coefficients of the polynomials can be obtained by solving a

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set of simultaneous algebraic equations resulted from the process of using the proposed method. In the subsequent sections we will illustrate the $(\frac{G'}{G})$-expansion method in detail with Eqs. (1.1), (1.2) and (1.3), (1.4).

2. Broer–Kaup equations

We begin with the Broer–Kaup equations (1.1), (1.2). In order to look for the travelling wave solutions of Eqs. (1.1), (1.2) we suppose that

$$u(x,t) = u(\xi), \quad v(x,t) = v(\xi), \quad \xi = x - Vt, \tag{2.1}$$

where the speed V of the travelling waves is to be determined later.

By using the travelling wave variable (2.1), Eqs. (1.1), (1.2) are converted into the ODEs for $u = u(\xi)$ and $v = v(\xi)$

$$-Vu' + uu' + v' = 0,$$
$$-Vv' + u' + (uv)' + u''' = 0.$$

Integrating the ODEs above with respect to ξ once yields

$$C_1 - Vu + \frac{1}{2}u^2 + v = 0, \tag{2.2}$$
$$C_2 - Vv + u + uv + u'' = 0, \tag{2.3}$$

where C_1 and C_2 are the integration constants that are to be determined later.

Considering the homogeneous balance between v and u^2 in Eq. (2.2) and that between u'' and uv in Eq. (2.3) $(2m = n, m + 2 = m + n \Rightarrow m = 1, n = 2)$, we suppose that the solution $(u(\xi), v(\xi))$ of Eqs. (2.2), (2.3) is of the form

$$u(\xi) = \alpha_1\left(\frac{G'}{G}\right) + \alpha_0, \quad \alpha_1 \neq 0, \tag{2.4}$$

$$v(\xi) = \beta_2\left(\frac{G'}{G}\right)^2 + \beta_1\left(\frac{G'}{G}\right) + \beta_0, \quad \beta_2 \neq 0, \tag{2.5}$$

where $G = G(\xi)$ satisfies the second order LODE

$$G'' + \lambda G' + \mu G = 0, \tag{2.6}$$

$\alpha_1, \alpha_0, \beta_2, \beta_1, \beta_0, \lambda$ and μ are constants to be determined later.

By using (2.4)–(2.6), it is derived that

$$u^2 = \alpha_1^2\left(\frac{G'}{G}\right)^2 + 2\alpha_1\alpha_0\left(\frac{G'}{G}\right) + \alpha_0^2,$$
$$uv = \alpha_1\beta_1\left(\frac{G'}{G}\right)^3 + (\alpha_0\beta_2 + \alpha_1\beta_1)\left(\frac{G'}{G}\right)^2 + (\alpha_1\beta_0 + \alpha_0\beta_1)\left(\frac{G'}{G}\right) + \alpha_0\beta_0,$$
$$u'' = 2\alpha_1\left(\frac{G'}{G}\right)^3 + 3\alpha_1\lambda\left(\frac{G'}{G}\right)^2 + (2\alpha_1\mu + \alpha_1\lambda^2)\left(\frac{G'}{G}\right) + \alpha_1\lambda\mu.$$

Substituting the expressions above into Eqs. (2.2), (2.3) and collecting all terms with the same power of $(\frac{G'}{G})$ together, the left hand sides of Eqs. (2.2), (2.3) are converted into the polynomials in $(\frac{G'}{G})$. Equating the coefficients of the polynomials to zero, yields a set of simultaneous algebraic equations for $\alpha_1, \alpha_0, \beta_2, \beta_1, \beta_0, \lambda, \mu, V, C_1$ and C_2 as follows:

0:	$C_1 - V\alpha_0 + \frac{1}{2}\alpha_0^2 + \beta_0 = 0,$
1:	$-V\alpha_1 + \alpha_1\alpha_0 + \beta_1 = 0,$
2:	$\frac{1}{2}\alpha_1^2 + \beta_2 = 0,$
0:	$C_2 - V\beta_0 + \alpha_0 + \alpha_0\beta_0 + \alpha_1\lambda\mu = 0,$
1:	$-V\beta_1 + \alpha_1 + \alpha_1\beta_0 + \alpha_0\beta_1 + 2\alpha_1\mu + \alpha_1\lambda^2 = 0,$
2:	$-V\beta_2 + \alpha_0\beta_2 + \alpha_1\beta_1 + 3\alpha_1\lambda = 0,$
3:	$\alpha_1\beta_2 + 2\alpha_1 = 0.$

Solving the algebraic equations yields

$$\begin{cases} \beta_2 = -2, \\ \beta_1 = -2\lambda, \\ \beta_0 = -(2\mu + 1), \\ \alpha_1 = \pm 2, \\ V = \alpha_0 \mp \lambda, \\ C_1 = \frac{1}{2}\alpha_0^2 \pm \lambda\alpha_0 + 2\mu + 1, \\ C_2 = \alpha_0 \pm \lambda \end{cases} \tag{2.7}$$

λ, μ and α_0 are arbitrary constants.