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Numerical solution for special non-linear Fredholm integral equation by HPM

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Abstract

The aim of this paper is to apply homotopy perturbation method (HPM) to solve a kind non-linear integral equation of Fredholm type. Two examples are presented to show the ability of the method. The results reveal that the method is very effective and simple.

Keywords: Homotopy perturbation method; Non-linear Fredholm integral equation

1. Introduction

Homotopy perturbation method introduced by He [1–4] has been used by many mathematicians and engineers to solve various functional equations. In this method the solution is considered as the sum of an infinite series which converges rapidly to the accurate solutions. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$ which is considered as a "small parameter". A considerable research works have been conducted recently in applying this method to a class of linear and non-linear equations. This method was further developed and improved by He and applied to non-linear oscillators with discontinuities [5], non-linear wave equations [6], boundary value problem [7], limit cycle and bifurcation of non-linear problems [8], and many other subjects [1–4]. It can by say that He's homotopy perturbation method is a universal one, is able to solve various kinds of non-linear functional equations. For examples it was applied to non-linear Schrödinger equations [9], to non-linear equations arising in heat transfer [10], to the quadratic Ricatti differential equation [11], and to other equations [12–15]. In [7], a comparison of (HPM) and homotopy analysis method was made, revealing that the former is more powerful than latter. We extend the method to solve non-linear Fredholm integral equation of second kind. Furthermore we will show that considerably better approximations related to the accuracy, level would be obtained. To demonstrate the above idea, numerical examples are given.

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2. Basic idea of He's homotopy perturbation method [1]

To illustrate the basic ideas of this method, we consider the following non-linear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{1}$$

with the boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma, \tag{2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω.

The operator A can be divided into two parts, which are L and N, where L is a linear, but N is non-linear. Eq. (1) can be, therefore, rewritten as follows:

$$L(u) + N(u) - f(r) = 0. \tag{3}$$

By the homotopy technique, we construct a homotopy $U(r,p) : \Omega \times [0,1] \to \mathbb{R}$, which satisfies:

$$H(U,p) = (1-p)[L(U) - L(u_0)] + p[A(U) - f(r)] = 0, \quad p \in [0,1], \ r \in \Omega, \tag{4}$$

or

$$H(U,p) = L(U) - L(u_0) + PL(u_0) + p[N(U) - f(r)] = 0, \tag{5}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is an initial approximation of Eq. (1), which satisfies the boundary conditions. Obviously, from Eqs. (4) and (5) we will have

$$H(U,0) = L(U) - L(u_0) = 0, \tag{6}$$

$$H(U,1) = A(U) - f(r) = 0. \tag{7}$$

The changing process of p form zero to unity is just that of $U(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called homotopy. According to the (HPM), we can first use the embedding parameter p as a small parameter, and assume that the solution of Eqs. (4) and (5) can be written as a power series in p:

$$U = U_0 + pU_1 + p^2U_2 + \cdots \tag{8}$$

Setting $p = 1$, results in the approximate solution of Eq. (1)

$$u = \lim_{p \to 1} U = U_0 + U_1 + U_2 + \cdots \tag{9}$$

The combination of the perturbation method and the homotopy method is called the homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantage of the traditional perturbation techniques. The series (9) is convergent for most cases. However the convergent rate date on non-linear operator $A(U)$, the following opinions are suggested by He [1]:

(1) The second derivative of $N(U)$with respect to U must be small because the parameter may be relatively large, i.e., $p \to 1$.
(2) The norm of $L^{-1}\frac{\partial N}{\partial V}$ must be smaller than one so that the series converges.

3. Method of solution

In this section we present homotopy perturbation method for solving a non-linear integral equation of Fredholm type:

$$\phi(x) = f(x) + \mu \int_a^b k(x,t)(\phi(t))^P \mathrm{d}t, \quad x \in [a,b], \ P \in N, \ P \geqslant 2, \tag{10}$$

where μ is a real number, the kernel $k(x,t)$ is a continuous function in $[a,b] \times [a,b]$ and $f(x)$ is a given continuous function defined in $[a,b]$.