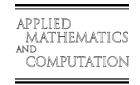


Applied Mathematics and Computation 202 (2008) 300-310



www.elsevier.com/locate/amc

Generalized Vandermonde determinants and mean values

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Abstract

In this article, the generalized Vandermonde determinants of order n + 1 are introduced and studied systematically, a type of mean values of several positive numbers are defined by using the same type of determinants, and some of their basic properties and applications are given.

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Keywords: Vandermonde determinants; Mean values; Integral identity; Elementary symmetric function; Monotonicity; Integral inequality

1. Introduction

Throughout the present investigation, we assume that

$$\mathbf{x} = (x_0, x_1, x_2, \dots, x_n), \quad \mathbb{N}_0 := \mathbb{N} \cup \{0\} \quad (\mathbb{N} := \{1, 2, 3, \dots\}),$$

and

$$\mathbb{R} = (-\infty, \infty), \quad \mathbb{R}_+ = [0, \infty), \quad \text{and} \quad \mathbb{R}_+^n = [0, \infty)^n.$$

The following determinant of the Vandermonde matrix of order n+1 is well known:

$$V(\mathbf{x}) := \begin{vmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ 1 & x_2 & x_2^2 & \dots & x_2^n \\ \dots & \dots & \dots & \dots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{vmatrix} = \prod_{0 \le i < j \le n} (x_j - x_i).$$

$$(1.1)$$

Obviously, if we set

$$x_0 < x_1 < \cdots < x_n$$

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then

$$V(\mathbf{x}) > 0.$$

Recently, Xiao and Zhang [3] gave the following general form of the Vandermonde determinant of order n + 1:

$$V(\mathbf{x};r) := \begin{vmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^{n-1} & x_0^{n+r} \\ 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} & x_1^{n+r} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} & x_2^{n+r} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} & x_n^{n+r} \end{vmatrix},$$
(1.2)

where

$$x_i \in \mathbb{R}_+ \quad (0 \le i \le n) \quad \text{and} \quad r \in \mathbb{R}.$$

In fact, Xiao and Zhang [3] also proved an integral identity relating V(x;r) with V(x) as follows:

$$V(\mathbf{x};r) = \prod_{k=1}^{n} (k+r) \cdot V(\mathbf{x}) \cdot \int_{E} \left(\sum_{i=0}^{n} x_{i} t_{i} \right)^{r} dt_{1} dt_{2} \cdots dt_{n},$$
(1.3)

where

$$t_0 = 1 - \sum_{i=1}^{n} t_i$$

and

$$E := \left\{ (t_1, t_2, \dots, t_n) : \sum_{i=1}^n t_i \leq 1 \mid (t_i \in \mathbb{R}_+; i = 1, \dots, n) \right\}.$$

Subsequently, Xiao et al. [4] studied the extended mean values E(p,q;x) by using the generalized Vandermonde determinant V(x;r) in (1.2) above and established the following result.

Theorem 1. Let

$$\mathbf{x} = (x_0, x_1, \dots, x_n) \in \mathbb{R}^{n+1}_{\perp} \quad (x_i \neq x_i \text{ when } i \neq j).$$

Then the extended mean values $E(p,q;\mathbf{x})$ of the (n+1)-tuple \mathbf{x} with two parameters p and q are increasing strictly with respect to both $p \in \mathbb{R}$ and $q \in \mathbb{R}$, where

$$E(p,q;\mathbf{x}) := \begin{cases} \left(\prod_{i=1}^{n} \frac{i+q}{i+p} \cdot \frac{V(\mathbf{x};p,0)}{V(\mathbf{x};q,0)}\right)^{1/(p-q)} & \left((p-q)\prod_{i=1}^{n}[(i+p)(i+q)] \neq 0\right), \\ \left(\frac{(-1)^{q+1}(-q-1)!(q+n)!}{\prod_{i=1}^{n}(i+p)} \cdot \frac{V(\mathbf{x};p,0)}{V(\mathbf{x};q,1)}\right)^{1/(p-q)} & (p \neq q; \ q \in \{-1,-2,\ldots,-n\}), \\ \left(\frac{(-1)^{q-p}(-q-1)!(q+n)!}{(-p-1)!(p+n)!} \cdot \frac{V(\mathbf{x};p,1)}{V(\mathbf{x};q,1)}\right)^{1/(p-q)} & (p \neq q; \ p,q \in \{-1,-2,\ldots,-n\}), \\ \exp\left(\frac{V(\mathbf{x};p,1)}{V(\mathbf{x};p,0)} - \sum_{i=1}^{n} \frac{1}{i+p}\right) & (p = q \notin \{-1,-2,\ldots,-n\}), \\ \exp\left(\frac{V(\mathbf{x};p,2)}{2V(\mathbf{x};p,1)} - \sum_{i=1}^{n} \frac{1}{i+p}\right) & (p = q; \ q \in \{-1,-2,\ldots,-n\}) \end{cases}$$

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