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On the motion of a second grade fluid due to longitudinal and torsional oscillations of a cylinder: A numerical study

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ABSTRACT

Unsteady problems involving the second grade fluids have received considerable attention in recent years. The present study is an attempt to look at the motion of an oscillating rod in a second grade fluid. Specifically, we solve numerically for the flow of a second grade fluid surrounding a solid cylindrical rod that is suddenly set into longitudinal and torsional motion. The equations are made dimensionless. The results are presented for the shear stresses at the wall, related to the drag force; these are physical quantities of interest, especially in oil-drilling applications.

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1. Introduction

One of the most challenging engineering problems encountered in off-shore oil drillings, towing operations, or excitations of long rods (or cables), is obtaining an accurate expression and estimate for the (viscous) drag, or the damping force, due to the fluid, exerted on the rod or cable. In oil-drilling applications, Akyildiz [1], for example, lists the cooling and lubricating of the bit and the drilling string as one of the important design and operational parameters, since to a large extent this depends on the complex rheological structure of the surrounding fluid which is composed of water, mud, oil, rocks, sediments, etc. In fact, as Dareing and Livesay [9] point out the performance of the drill string under the dynamic conditions, i.e., longitudinal and torsional (angular) oscillations, is significantly influenced by the viscous (frictional) force of the fluid, which in turn can cause (thermal) stresses in the string which can affect the drilling rate and hole stability. The common practice has been to assume a linear relationship between the external damping force and the velocity of the rod, but as indicated by Caserella and Laura [8] this approach is not very accurate.

Interest and research activities in drag-reduction techniques, whether in transportation systems (on ground, in or under water, or in air), oil-drilling industries, or materials handling have increased in the last few decades. Bushnell and Moore [6] describe that the various attempts to study these problems can generally be classified into (at least) three categories: (i) form-drag reduction, (ii) skin-friction drag reduction, and (iii) drag-due-to-lift reduction. Amongst the techniques commonly used in many chemical industries to reduce the skin friction drag is to use (surface) additives. It has also long been known that some additives can change the rheological properties of Newtonian fluids, such as water, to non-Newtonian fluids and that a significant reduction in drag of such non-Newtonian fluids flowing past solid objects has been observed (see [51,24,23]). As a result, in recent years, there have been many studies concerned with the calculation of the wall shear stresses and the drag for the flow of various non-Newtonian fluids past solid objects such as spheres and cylinders. The flow of a viscous fluid past a sphere was first studied by Stokes; for non-Newtonian fluids there have also been many such studies (see [19–21,18]).

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The motion of an oscillating cylinder and its effects on the surrounding fluid was first investigated by Stokes [54] who obtained an exact solution for the rotational oscillations of an infinite cylindrical rod immersed in a linear viscous fluid. This work has been extended and with the publications of the papers by Casarella and Laura [8] and Rajagopal [36] on an oscillating cylindrical rod, a host of other papers have appeared in the literature: Rajagopal et al. [40] studied the same problem with the cylinder oscillating in a simple fluid. Ramkissoon et al. [49] studied the flow due to the longitudinal and torsional motion of a rod in an unbounded micropolar fluid (see also [48]). Maneschy and Massoudi [26] obtained analytical expressions for the shear stresses acting on the surface of a cylindrical rod undergoing longitudinal and torsional oscillations in a second grade fluid. Bandelli et al. [3] studied the same problem where the fluid was modeled as a third grade fluid; they obtained exact solutions for the case of a rotating rod and also presented a perturbation solution for the case of a rod undergoing both longitudinal and torsional oscillations. Pontrelli [32], based on the results of Rajagopal and Bhatnagar [42], studied the motion of an Oldroyd-B fluid due to the longitudinal and torsional oscillations of an infinite porous cylinder, with suction or injection at the surface. Akyildiz [1] studied a similar problem and discussed the effects of Weissenberg number (related to the elasticity of the fluid) and the viscosity ratio on the flowfield. In all the cases mentioned so far, it is assumed that the cylinder is immersed in a fluid. It is, however, also possible to study the internal flow, i.e., flow inside a cylinder subjected to oscillations. Calmelet-Eluhu and Majumdar [7] studied this problem for the case of a micropolar fluid, while Fetecau [14], Fetecau and Fetecau [15] presented some interesting unsteady solutions of certain non-Newtonian fluids such as the second grade and Oldroyd-B fluids in cylindrical geometries. In this paper, we re-consider the problem studied by Maneschy and Massoudi [26] and present numerical solutions to the full equations of motion of a second grade fluid due to torsional and longitudinal oscillations of an infinitely long circular cylinder with constant cross sectional area.

2. Governing equations

The governing equations of motion, where there are no thermal, chemical, or electromagnetic effects, are the conservation of mass and linear momentum. These are

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{u}) = 0, \tag{1a}$$

where ρ is the density of the fluid, $\partial/\partial t$ is the partial derivative with respect to time, and $\mathbf{u}$ is the velocity vector. For an isochoric motion we have

$$\mathrm{div}\mathbf{u} = 0. \tag{1b}$$

Conservation of linear momentum:

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathrm{div}\mathbf{T} + \rho \mathbf{b}, \tag{2}$$

where $\mathbf{b}$ is the body force vector, $\mathbf{T}$ is the stress tensor, and $\mathrm{d}/\mathrm{d}t$ is the total time derivative, given by

$$\frac{\mathrm{d}(.)}{\mathrm{d}t} = \frac{\partial(.)}{\partial t} + [\mathrm{grad}(.)]\mathbf{u}. \tag{3}$$

3. Constitutive relation

Perhaps the simplest model which can capture the normal stress effects (which could lead to phenomena such as 'die-swell' and 'rod-climbing', which are manifestations of the stresses that develop orthogonal to planes of shear) is the second grade fluid, or the Rivlin–Ericksen fluid of grade two [50,55]. This model has been used and studied extensively [10] and is a special case of fluids of differential type [11]. For a second grade fluid the Cauchy stress tensor is given by

$$\mathbf{T} = -p\mathbf{1} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2, \tag{4}$$

where p is the indeterminate part of the stress due to the constraint of incompressibility, μ is the coefficient of viscosity, α_1 and α_2 are material moduli which are commonly referred to as the normal stress coefficients. The kinematical tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined through

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^{\mathrm{T}}, \tag{5}$$

$$\mathbf{A}_2 = \frac{\mathrm{d}\mathbf{A}_1}{\mathrm{d}t} + \mathbf{A}_1 \mathbf{L} + (\mathbf{L})^{\mathrm{T}} \mathbf{A}_1, \tag{6}$$

$$\mathbf{L} = \mathrm{grad}\mathbf{u}. \tag{7}$$