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Emission policy in an economic union with Poisson technological change

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ABSTRACT

This study examines optimal emission policy in a union of countries. In each country, labor is allocated between production and R&D which generates Poisson technological change. Production incurs emissions that are spread all over the union and aggravates pollution. Utility in any country depends negatively on both emissions and pollution in the union. The central planner of the union sets emission taxes for the countries. This study constructs a Pareto-optimal emission policy for the union.

1. Introduction

In this study, I examine the coordination of emission policy in a union of countries. The production of goods in any country incurs emissions that are spread all over the union, but efficiency in production can be improved by research and development (R&D). The outcome of R&D is random and follows a Poisson process. Welfare in any country depends positively on the level of consumption and negatively on total emissions in the union. The central planner of the union controls emissions by union-wide taxes. The purpose for study is to find out the Pareto-optimal emission taxes for the countries.

The impact of any environmental policy depends crucially on the existence of uncertainty. The papers [4,8,10] and [11] consider public policy by a growth model where productivity shocks follows a Wiener process. Soretz [9] applies that approach to environmental policy. In contrast, I assume that uncertainty is directly embodied in technological change in the form of Poisson processes.

Beltratti et al. [3] introduce a growth model where an environmental asset is a source of utility and depleted by a pollution process which is linked to consumption. They define the concept of the Green Golden rule as the best sustainable configuration, i.e. the path that gives the highest maintainable level of instantaneous utility. Ayong Le Kama [2] transforms this model by linking the pollution process to production. Following these papers, I search for the Green Golden Rule for the economic union.

I assume that in each country there is a local planner that maximizes welfare and has enough instrument to control the allocation of resources in the country. The local planners of all countries realize that decisions in one country affect welfare in the other countries through emissions and pollution. Following Dixit [5], I assume that each local planner estimates the response of the others and makes its decisions by this information. To construct a Pareto optimal policy for the union, I introduce a central planner that maximizes welfare in the union through influencing the local planners' decisions by emission taxes.

This paper is organized as follows. Sections 2 and 3 present the general structure of the union and a single country. Section 4 examines a local planner's and Section 5 the central planner's behavior. The optimization problems are solved by dynamic programming with Poisson jump processes (technological change) and an ordinary differential equation (the accumulation of pollution) as constraints.

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2. The union

I consider a union of fixed number n of similar countries. In country $j \in \{1, \ldots, n\}$, there is a fixed labor supply L, of which the amount l_j is used in production and the rest z_j in R&D:

$$L = l_j + z_j. \tag{1}$$

Emissions m_j are in fixed proportion to labor input in production, l_j, in each country. By a proper choice of the unit of labor, local emissions m_j and total emissions in the whole union, m, are then given by

$$m_j = l_j, \quad m \doteq \sum_{j=1}^{n} m_j = \sum_{j=1}^{n} l_j. \tag{2}$$

Each country $j \in \{1, \ldots, n\}$ produces a different good. In the union, competitive firms produce a consumption good from all these goods according to

$$\sum_{j=1}^{n} c_j = y = n \prod_{j=1}^{n} y_j^{1/n}, \tag{3}$$

where c_j is consumption in country j, y_j output in country j, and y total consumption in the union. With some complication, the same results can be generalized for any neoclassical production function with constant returns to scale. Let p_j be the price for good j. With Cobb–Douglas technology (3), the consumption price p is obtained by minimizing the unit cost $\frac{1}{y}\sum_{j=1}^{n} p_j y_j = \sum_{j=1}^{n} p_j \frac{y_j}{y}$ of the consumption good by the input–output ratios $(y_1/y, \ldots, y_n/y)$:

$$p = \min_{y_1/y, \ldots, y_n/y} \left\{ \sum_{j=1}^{n} p_j \frac{y_j}{y} \middle| \prod_{j=1}^{n} \left(\frac{y_j}{y}\right)^{1/n} = \frac{1}{n} \right\} = \prod_{j=1}^{n} p_j^{1/n}.$$

Because in the model there is no money that would pin down the nominal price level at any time, the consumption price p can be normalized at unity:

$$1 = p = \prod_{j=1}^{n} p_j^{1/n}. \tag{4}$$

Let t be time and P be the level of pollution in the union. Following [7], I assume that total emissions m contribute the degree of pollution, P, but the nature absorbs pollution at a constant rate h:

$$\dot{P} \doteq \mathrm{d}P/\mathrm{d}t = m - hP. \tag{5}$$

3. The countries

Assume that all planners share the same preferences and that total emissions and the degree of pollution in the union decrease welfare in all countries. In country j, the utility from an infinite stream of its consumption c_j, emissions m and pollution P beginning at time T is then given by

$$E \int_{T}^{\infty} u(c_j, m, P) \mathrm{e}^{-\rho(t-T)} \mathrm{d}t, \quad \frac{\partial u}{\partial c_j} > 0, \quad \frac{\partial u}{\partial m} < 0, \quad \frac{\partial u}{\partial P} < 0, \tag{6}$$

where E is the expectation operator, $\rho > 0$ the constant rate of time preference and u the level of instantaneous utility. Because it is impossible to find any analytical solution for the general case (6) in Bellman's dynamic programming, I specify the instantaneous utility function in the exponential form

$$u(c_j, m, P) \doteq c_j^{\sigma} m^{-\delta} P^{-\nu}, \quad 0 < \sigma < 1, \quad \delta > 0, \quad \nu \geqslant 0, \tag{7}$$

where σ, δ and ν are constants. The constant $1 - \sigma \in (0, 1)$ is the constant rate of risk aversion. Following [2] and [3], I make the following definition:

Definition. The *Green Golden rule* (*GGR*) is the pattern of development that gives the highest maintainable level of instantaneous utility (7) for all countries $j \in \{1, \ldots, n\}$.

The efficiency of labor l_j in production in country j is A^{γ_j}, where $A > 1$ is a constant and γ_j is the serial number of technology. In the advent of technological change in country j, this efficiency increases from A^{γ_j} to A^{γ_j+1}. Total output in country j is given by

$$y_j = A^{\gamma_j} l_j. \tag{8}$$

In production, firms employ labor l_j up to the point where the wage w_j is equal to the output price p_j times the marginal product of labor, $\partial y_j / \partial l_j$: