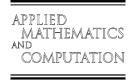




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# On quadrature formulae based on derivative collocation

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#### Abstract

The recent article "A new type of weighted quadrature rules and its relation to orthogonal polynomials" by Masied-Jamei [M. Masjed-Jamei, A new type of weighted quadrature rules and its relation to orthogonal polynomials, Appl. Math. Comput. 188 (2007) 154–165] introduces quadrature rules based on the evaluation of the derivative(s) of the integrand function rather the function itself. The approach appears useful when a number of derivatives, including the integrand, vanish at a point  $\lambda$ , leading to increased order of accuracy compared to standard Gaussian rules. It is also shown by Masjed-Jamei (2007) how the nodes and weights of the resulting quadrature formula relate to nodes and weights of standard Gaussian quadratures applied to a weight function w to be determined by solving a specific system of integral equations. We give here an explicit expression for w and provide strategies for the practical computation of the quadrature nodes and weights. Additional comments on the examples used by Masjed-Jamei (2007) as well as a generalization involving multiple  $\lambda$ 's, are also included.

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#### 1. Introduction

In [5], Masied-Jamei introduces quadrature rules of the form,

$$\int_{\beta}^{\alpha} g(x)\rho(x) \, \mathrm{d}x \approx \sum_{i=1}^{N} \rho_{i} f^{(m)}(x_{i}),\tag{1}$$

where

$$g(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(\lambda) \frac{(x-\lambda)^k}{k!}$$
 (2)

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is the difference between f and its Taylor polynomial of degree (at most) m-1 at some value  $\lambda$ . Note that

- $g(\lambda) = \cdots = g^{(m-1)}(\lambda) = 0$  and
- (1) is automatically satisfied for  $f(x)=1,(x-\lambda),\ldots,(x-\lambda)^{(m-1)}$ .

In particular, the quadrature (1) can be expected to be exact for polynomials of degree 2N + m - 1 upon appropriately selecting its nodes and weights. This is not entirely surprising, since (1) can be restated as a rule

$$\int_{\alpha}^{\beta} f(x)\rho(x)dx \approx \sum_{k=0}^{m-1} \tilde{\rho}_k f^{(k)}(\lambda) + \sum_{i=1}^{N} \rho_i f^{(m)}(x_i)$$

with N nodes and N+m weights to be determined. Substituting  $f(x)=(x-\lambda)^k$  shows that the weights  $\tilde{\rho}_k, k=0,\ldots,m-1$ , can be expressed explicitly in terms of the basic moments of order up to m-1 at  $\lambda$ :

$$\tilde{\rho}_k \equiv \frac{\mu_k}{k!}, \quad \mu_k \equiv \int_{\alpha} \beta(x - \lambda)^k \rho(x) dx.$$
 (3)

It is shown in [5] how the remaining nodes and weights can be obtained from standard Gaussian quadratures on an interval [a,b] (to be determined) by solving integral equations

$$\int_{a}^{b} (t - \lambda)^{j} w(t) dt = \frac{j!}{(j + m)!} \int_{a}^{\beta} (x - \lambda)^{j + m} \rho(x) dx, \quad j = 0, \dots, 2N - 1$$
(4)

for a weight function w (compare [5, Eq. (25)]). Provided [a, b] and w can be found such that (4) holds, the nodes and weights of the quadrature (1) are then simply given by the nodes  $x_j = t_j$  and weights  $\rho_j$  of the Gaussian quadrature,

$$\int_{a}^{b} f(t)w(t)dt \approx \sum_{j=1}^{N} \rho_{j}f(t_{j}).$$
(5)

In this note, we show that a weight function w can in fact be explicitly determined in terms of  $\rho$ , using the Peano kernel representation of the function g itself (in [5] Masjed-Jamei uses such representation in the error analysis, but missed the connection in the derivation of the rule, despite coming close in [5, Section 2.1], when commenting on possible applications). We show that w is definite when  $\lambda$  in outside the (open) interval of integration ( $\alpha$ ,  $\beta$ ) (Section 2) but indefinite (changes sign) when  $\alpha < \lambda < \beta$  and m is odd (Section 3). We nevertheless show that the only example of this type considered in [5] can be reformulated, because of symmetry, as a rule (1) for which  $\lambda = \alpha$ . In Section 4 we comment on the practical determination of the nodes and weights, which is not addressed in [5]. Finally, additional issues relevant to the quadrature (1) are included in Section 5. In particular, a generalization of (1), where g vanishes at m distinct points is presented.

#### 2. Definite case: $\lambda \not\in (\alpha, \beta)$

W.l.o.g. assume  $\lambda \le \alpha$  (otherwise let  $x \to -x$  in (1)). The Peano kernel representation of the linear functional  $f \to g$  yields

$$g(x) = \int_{a}^{x} \frac{(x-t)^{m-1}}{(m-1)!} f^{(m)}(t) dt,$$
(6)

which can be shown via successive integration by parts. Therefore, the integral on the left-hand side of (1) can be written as

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