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Global asymptotic stability of stochastic recurrent neural networks with multiple discrete delays and unbounded distributed delays ☆

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ABSTRACT

In this paper using Lyapunov–Krasovskii functional and the linear matrix inequality (LMI) approach the global asymptotic stability of stochastic recurrent neural networks with multiple discrete time-varying delays and distributed delays is analyzed. A new sufficient condition ensuring the global asymptotic stability for delayed recurrent neural networks is obtained in the stochastic sense using the powerful MATLAB LMI toolbox. Two examples are provided to illustrate the applicability of the stability results.

1. Introduction

The dynamics of neural networks have been extensively studied in recent years because of their application in many areas such as associative memory, pattern recognition and optimization [8–10,14]. Many researchers have a lot of contributions to these subjects. Stability is a basic knowledge for dynamical systems and is useful in the application to the real life systems. The time delays are commonly encountered in various engineering systems such as chemical processes, hydraulic and rolling mill systems, etc. In fact, the stability analysis issue for recurrent neural networks with time delays has been an attractive subject of research in the past few years, where the time delays under consideration can be classified as constant delays, time-varying delays, and distributed delays. Various sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the global asymptotic or exponential stability for the recurrent neural networks with time delays, see example [3–5,7]. In some of the recent publications even though many methods have been exploited, the LMI approach and M-matrix approach have treated as the emerging methods to study the stability results.

The linear matrix inequality (LMI) technique has been extensively applied to tackle various stability problems of neural networks and stabilization problems of control systems. The advantages of the stability results based on LMI include that not only they are easily verified using the interior-point algorithms, but also they consider the neuron's inhibitory and excitatory effects on neural networks.

Most of the works on delayed neural networks have dealt with the stability analysis problems for neural networks with discrete delays. Neural networks has a spatial nature due to the presence of parallel pathways with a variety of axon sizes and lengths, so it is desirable to model them by introducing unbounded delays. In recent years there has been a growing research interest in the study of neural networks with distributed delays [16,17,20,22,24,25]. It should be mentioned that using linear matrix inequality (LMI) approach the sufficient global asymptotic stability conditions have been derived in

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[20,22] for a general class of neural networks with both discrete and distributed delays. Very recently, Zhang et al. [25] studied global exponential stability for non-autonomous cellular neural networks with unbounded delays. In real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that a neural network could be stabilized or destabilized by certain stochastic inputs [1]. Hence, the stability analysis problem for stochastic neural networks becomes increasingly significant, and some results related to this problem have recently been published, see [1,11,13,18]. So far, there are only a few papers that have taken stochastic phenomenon into account in neural networks [1,15,18]. Practically, such phenomenon always appears in the electrical circuit design of neural networks. Wang et al. [19,23] were studied the exponential stability of uncertain stochastic neural networks with discrete and distributed delays and robust stability for stochastic Hopfield neural networks with time delays. Huang and Cao [12] studied the exponential stability of uncertain stochastic neural networks with multiple time-varying delays in terms of LMI.

Based on the above discussions, a class of stochastic recurrent neural networks with both multiple time-varying discrete delays and unbounded distributed delays is considered in this paper. The main purpose of this paper is to study the global asymptotic stability in the mean square for stochastic recurrent neural networks with both multiple time-varying discrete delays and unbounded distributed delays. To the best of the authors knowledge there were no global stability results for stochastic recurrent neural networks with both multiple time-varying discrete delays and unbounded distributed delays. By using Lyapunov–Krasovskii functional we obtain the sufficient conditions for global asymptotic stability in the mean square for stochastic recurrent neural networks in terms of linear matrix inequality (LMI), which can be easily calculated by MATLAB LMI toolbox. We also provide two numerical examples to demonstrate the effectiveness of the proposed stability results.

2. Problem description and preliminaries

Throughout the manuscript we will use the notation $A > 0$ (or $A < 0$) to denote that the matrix A is a symmetric and positive definite (or negative definite) matrix. The notation A^{T} and A^{-1} mean the transpose of A and the inverse of a square matrix. If A, B are symmetric matrices $A > B$ $(A \geqslant B)$ means that $A - B$ is positive definite (positive semi-definite).

Consider the following neural networks with multiple discrete time-varying and unbounded distributed delays can be described by the integro-differential equations

$$\begin{aligned} x_i'(t) &= -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}^{(k)} f_j(x_j(t-\tau_k(t))) + \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\mathrm{d}s + I_i, \\ &i = 1,2,\ldots,n, \quad k = 1,2,\ldots,r, \end{aligned} \tag{1}$$

where $x_i(t)$ is the state of the ith neuron at time t, $a_i > 0$ denotes the passive decay rate, b_{ij}, $b_{ij}^{(k)}$ and c_{ij} are the synaptic connection strengths, f_j denotes the neuron activations, I_i is the constant input from outside the system, $\tau_k(t)$ represents the discrete transmission delay with $\dot{\tau}_k(t) \leqslant \eta_k < 1$ and $\tau^* = \max\{\tau_k(t)\}$ and the delay kernel k_j is a real valued continuous function defined on $[0, +\infty]$ and satisfies, for each i

$$\int_0^{\infty} k_j(s)\mathrm{d}s = 1. \tag{2}$$

We assume that the neuron activation functions f_j, $j = 1,2,\ldots,n$ satisfy the following hypotheses:

(H1) $|f_j(\zeta_1) - f_j(\zeta_2)| \leqslant L_j|\zeta_1 - \zeta_2|$ for all $\zeta_1, \zeta_2 \in \mathbb{R}$, $\zeta_1 \neq \zeta_2$.
(H2) f_j is bounded function for any $j = 1,2,\ldots,n$.
(H3) $0 \leqslant |f_j(\zeta_1) - f_j(\zeta_2)| \leqslant L_j|\zeta_1 - \zeta_2|$ for all $\zeta_1, \zeta_2 \in \mathbb{R}$, $\zeta_1 \neq \zeta_2$.

Assume that $x^* = (x_1^*, x_2^*, \ldots x_n^*)^{\mathrm{T}}$ is an equilibrium point of Eq. (1). It can be easily verify that the transformation $y_i = x_i - x_i^*$ transforms system (1) into the following system:

$$y'(t) = -Ay(t) + Bg(y(t)) + \sum_{k=1}^{r} B^{(k)} g(y(t-\tau_k(t))) + C\int_{-\infty}^{t} K(t-s) g(y(s))\mathrm{d}s, \tag{3}$$

where $y = [y_1, y_2, \ldots, y_n]^{\mathrm{T}}$, $A = \operatorname{diag}[a_1, a_2, \ldots, a_n]$, $B = [b_{ij}]$, $C = [c_{ij}]$, $D = [d_{ij}]$, $K(t-s) = \operatorname{diag}[k_1(t-s), k_2(t-s), \ldots, k_n(t-s)]$, $g(y) = [g_1(y_1), g_2(y_2), \ldots, g_n(y_n)]^{\mathrm{T}}$ with $g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*)$. Note that since each function $f_j(\cdot)$ satisfies the hypothesis (H2)–(H3), hence each $g_j(\cdot)$ satisfies

$$\begin{aligned} &g_j^2(\zeta_j) \leqslant L_j^2 \zeta_j^2, \\ &\zeta_j g_j(\zeta_j) \geqslant \frac{g_j^2(\zeta_j)}{L_j} \quad \forall \zeta_j \in \mathbb{R}, \\ &g_j(0) = 0. \end{aligned}$$