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Fixed-bed drying simulation of agricultural products using a new backward finite difference scheme

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Abstract

This work is concerned with the numerical simulation of fixed-bed corn drying using MSU (Michigan State University) drying model. The classical numerical procedure for MSU model relies on an explicit method of finite differences which requires certain stability conditions between the step sizes of the time and space variables. The objective of the present paper is to establish a stable implicit method based on backward finite differences, in both time and space variables, which takes into account some specific empirical aspects of the problem. Computational results illustrate the efficiency and the flexibility of method.

Keywords: Numerical methods; Finite difference; Drying model

1. Introduction

The preservation and commercialization of agricultural products, leading to full commercialization, demand appropriate storage which largely depends on the amount of water inside the products. Preservation begins with a drying stage in order to reduce the product moisture content to levels that guarantee its durability for long periods of time. Drying is a crucial stage and there is consensus that, when inappropriate, it is an important cause of deterioration of the agricultural products. Besides that, drying is the stage where the largest amount of energy is consumed, which could reach up to 60% (cf. Brooker et al. [2]) of the amount spent in the production chain. The mathematical modeling that describes the physical phenomenon of drying is intended to predict results for the product moisture content close to what is commercially expected. It is a simultaneous

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heat and mass transfer process between the product, which looses moisture, and the air, which gives energy to the process.

The MSU (Michigan State University) drying model is composed by a system of partial differential equations involving four variables, the moisture and temperature of the product, the temperature and the humidity of the drying air. This model is usually solved with forward explicit finite differences methods. In this work, it is proposed a new numeric procedure, that solves the system of equations using an implicit finite difference procedure. The consistence of the numerical simulations for drying is made by the analysis of results, which consists in the reduction of the product moisture content and the increase in its temperature, as well as, in the reduction of the temperature of the air and in the increase of its absolute humidity. The present work has, therefore, the following objectives:

1 – Simulate numerically the drying of agricultural products in fixed beds, using the MSU model.
2 – Establish a backward finite difference scheme to solve numerically the associated system of partial differential equations.

2. Material and method

The MSU fixed-bed drying model is composed by the balance of drying air enthalpy, the balance of energy for the agricultural product, the balance of mass for the drying air, and the balance of mass for the agricultural product. For all this, it is considered a control volume ($A\mathrm{d}x$) of area A and a differential height $\mathrm{d}x$, in an arbitrary position of the layer of the product. The model is discussed in Brooker et al. [2], and it is represented by the following system of partial differential equations:

$$\begin{cases} \dfrac{\partial T}{\partial x} = \dfrac{-ha(T-\theta)}{G_\mathrm{a}C_\mathrm{a} + G_\mathrm{a}C_\mathrm{v}W}, \\ \dfrac{\partial \theta}{\partial t} = \dfrac{ha(T-\theta)}{\rho_\mathrm{p}C_\mathrm{p} + \rho_\mathrm{p}C_\mathrm{w}U} - \dfrac{h_\mathrm{fg} + C_\mathrm{v}(T-\theta)}{\rho_\mathrm{p}C_\mathrm{p} + \rho_\mathrm{p}C_\mathrm{w}U} G_\mathrm{a} \dfrac{\partial W}{\partial x}, \\ \dfrac{\partial W}{\partial x} = \dfrac{\rho_\mathrm{p}}{G_\mathrm{a}} \dfrac{\partial U}{\partial t}, \\ \dfrac{\partial U}{\partial t} = \text{an empirical thin-layer equation}, \end{cases} \tag{2.1}$$

with the initial and boundary conditions:

$$\begin{cases} T(0,t) = T_0, \\ \theta(x,0) = \theta_0, \\ W(0,t) = W_0, \\ U(x,0) = U_0. \end{cases} \tag{2.2}$$

The definition of the parameters of the system are as follows: x – abscissa of the material point in the direction x (m); t – time (h); T – temperature of the air, function of x and t (C); W – absolute humidity of the air, function of x and t ($\mathrm{kg_v\ kg_{as}^{-1}}$); θ – temperature of the product, function of x and t (C); U – moisture content of the product, function of x and t (non-dimensional dry basis); h – convection heat transfer coefficient ($\mathrm{J\ m^{-2}\ C^{-1}\ h^{-1}}$); a – surface area of the particles per unit volume ($\mathrm{m^2\ m^{-3}}$); G_a – airflow ($\mathrm{kg_{da}\ h^{-1}\ m^{-2}}$); C_a – specific heat of the dry air ($\mathrm{J\ kg^{-1}\ C^{-1}}$); C_v – specific heat of the water vapor ($\mathrm{J\ kg^{-1}\ C^{-1}}$); ρ – bulk density of the product ($\mathrm{kg_{dm}\ m^{-3}}$); C_p – specific heat of the product ($\mathrm{J\ kg^{-1}\ C^{-1}}$); C_w – specific heat of the water in liquid state ($\mathrm{J\ kg^{-1}\ C^{-1}}$); h_fg – heat of evaporation ($\mathrm{J\ kg^{-1}}$); T_0 – initial temperature of the drying air (C); W_0 – absolute humidity of the drying air ($\mathrm{kg_v\ kg_{as}^{-1}}$); θ_0 – initial temperature of the product (C); U_0 – initial humidity of the product (dry basis).

Now the system (2.1) can be rewritten taking into account some specific empirical assumptions. Using Newton's law applied to mass transfer for the fourth equation in (2.1), one obtains: