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Global solutions for a nonlinear wave equation

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Abstract

In this work the existence of a global solution for the mixed problem associated to the nonlinear equation

$$u'' + M(|A^{\frac{1}{2}}u|^2)Au + N(|A^{\alpha}u|^2)A^{\alpha}u' = f$$

is proved in a Hilbert space framework by using Galerkin method.

Keywords: Nonlinear wave equation; global solution; Galerkin method

1. Introduction

The equation

$$u_{tt} - M\left(\int_{\Omega} |\nabla u|^2 \,\mathrm{d}x\right)\Delta u = f \tag{1}$$

is a generalization of the nonlinear wave equation proposed by Kirchhoff [11]. When $M = 1$ it becomes the linear wave equation

$$u_{tt} - \Delta u = f.$$

Eq. (1) has been studied by several authors. Global solutions to the Cauchy problem were obtained by considering analytical initial data and some growth properties for M. Generalizations of Eq. (1) are mainly characterized by the presence of new terms modelling restoring forces, dampings or sources. The abstract version of Eq. (1) is

$$u'' - M(|A^{\frac{1}{2}}u|^2)Au = f, \tag{2}$$

where A is an operator defined in a real Hilbert space H.

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In 1991, Hosoya and Yamada [4] studied the problem

$$\begin{cases} u_{tt} - M\left(\int_\Omega |\nabla u|^2 \,dx\right)\Delta u + \delta|u|^\alpha u = f, & x \in \Omega, \quad 0 < t < T, \\ u = 0, & x \in \partial\Omega, \quad 0 < t < T, \\ u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x), & x \in \Omega, \\ \delta > 0, \quad \alpha \geqslant 0, \quad T > 0. \end{cases}$$

They proved the existence and uniqueness of a strong local solution by using Galerkin method.

In 1990, Medeiros and Milla Miranda [14] proved the existence of global solutions and exponential decay of the energy for the abstract equation

$$u'' - M(|A^{\frac{1}{2}}u|^2)Au + A^\alpha u' = f,$$

where $\alpha \in]0,1]$. Uniqueness was obtained for $\frac{1}{2} < \alpha \leqslant 1$.

In 1991, Ikehata [5] studied the local solvability of an abstract initial value problem of the form

$$(P_\delta)\begin{cases} u''(t) - M(|A^{\frac{1}{2}}u(t)|^2)Au(t) + \delta u'(t) = f(u(t)) \quad \text{in } [0,T], \\ u(0) = u_0 \quad u'(0) = u_1, \end{cases}$$

where A is a positive self-adjoint operator in H, f is a nonlinear operator from $D(A^{\frac{1}{2}})$ to $H, \delta \in \mathbb{R}$ and $M \in C^1[0,\infty)$ satisfies $M(s) \geqslant m_0 > 0$, with m_0 constant. The existence of a strong local solution was obtained with arbitrary initial data and without compactness hypothesis. Ikehata used the Theory of Kato (cf. [8,9]). In [5], the author also considered the blowing-up of the solution when $\delta = 0$. The result is then applied to the mixed problem associated to the equation

$$u_{tt}(x,t) - M\left(\int_\Omega |\nabla u(y,t)|^2 \,dy\right)\Delta u(x,t) = |u(x,t)|^2 u(x,t).$$

In 1992, Ikehata and Okazawa [7] studied the Cauchy problem for the quasilinear evolution equation

$$\begin{cases} u''(t) - M(|A^{\frac{1}{2}}u(t)|^2)Au = f(u(t)), \quad t \geqslant 0, \\ u(0) = u_0, \quad u'(0) = u_1. \end{cases}$$

Existence of a strong local solution was obtained by using again the theory of Kato.

In 1996, Matsuyama and Ikehata [6] studied the associated mixed problem to the equation

$$u_{tt} - M(\|\nabla u\|^2)\Delta u + \delta|u_t|^{p-1}u_t = \mu|u_t|^{q-1}u_t,$$

and showed the existence of a global solution and decay of the associated energy when $t \to +\infty$, with some restrictions to the initial data.

In this line of generalizations, in 1996, the present author [3], considered the local solvability of the problem

$$\begin{cases} u'' - M(\|\nabla u\|^2)\Delta u + N(\|u_t\|^2)u_t = b|u|^{p-1}u, & \text{in } \Omega \times (0,T_0), \\ u = u_0, \quad u_t = u_1, & \text{in } \Omega \times \{0\}, \\ u = 0, & \text{in } \partial\Omega \times (0,T_0), \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with regular boundary $\partial\Omega$ and b is a positive constant. Here $\|\cdot\|$ denotes the usual L^2-norm. Also, in 1998, Cavalcanti et al. [2], proves the existence of global solution and exponential decay of a generalized wave equation. In analogous form, in 1999, Lasiecka and Ong [12], study the global solvability of quasi linear equation with nonlinear boundary dissipation and study too the uniform decay of solutions. Also, it has relevant importance for the theme the work of Perla Mezala [15] on the classical solutions of a quasi linear hyperbolic equation and of the Sideris [16] on the formation of singularities in solutions to nonlinear hyperbolic equations.

In the present work the problem

$$\begin{cases} u'' + M(|A^{\frac{1}{2}}u|^2)Au + N(|A^\alpha u|^2)A^\alpha u' = f, \\ u = u_0 \quad u_t = u_1 \end{cases}$$

is studied, where A is a positive self-adjoint operator defined in H and the functions M and N satisfy certain properties.