Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 194 (2007) 534–539
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Two algorithms for solving a general backward tridiagonal linear systems

A.A. Karawia

Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

Abstract

In this paper we present an efficient computational and symbolic algorithms for solving a backward tridiagonal linear systems. The implementation of the algorithm using Computer algebra systems (CAS) such as Maple, Macsyma, Mathematica, and Matlab is straightforward. An examples are given in order to illustrate the algorithms.

Keywords: Backward tridiagonal matrices; Tridiagonal matrices; Linear systems; Determinants; Computer algebra systems (CAS)

1. Introduction

Many problems in mathematics and applied science require the solution of linear systems having a backward tridiagonal coefficient matrices. This kind of linear system arise in many fields of numerical computation [1,2].

The main goal of the current paper is to develop an efficient algorithms for solving a general backward tridiagonal linear systems of the form:

$$AX = Y, \tag{1.1}$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & a_1 & d_1 \\ 0 & 0 & 0 & \cdots & a_2 & d_2 & b_2 \\ 0 & 0 & 0 & \cdots & d_3 & b_3 & 0 \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ a_{n-1} & d_{n-1} & b_{n-1} & \cdots & 0 & 0 & 0 \\ d_n & b_n & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}, \tag{1.2}$$

$X = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$, $Y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}}$ and $n \geqslant 3$.

E-mail address: abibka@mans.edu.eg

0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.04.057

A general $n \times n$ backward tridiagonal matrix A of the form (1.2) can be stored in $3n-2$ memory locations by using three vectors $\mathbf{a} = (a_1, a_2, \ldots, a_{n-1})$, $\mathbf{b} = (b_2, b_3, \ldots, b_n)$, and $\mathbf{d} = (d_1, d_2, \ldots, d_n)$. When considering the system (1.1) it is advantageous to introduce one additional vector $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_n)$. This vector is related to the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{d}$.

The current paper is organized as follows. In Section 2, the main results are given. An illustrative examples are presented in Section 3. In Section 4, a Conclusion is given.

2. Main results

In this section we are going to formulate a modified computational algorithm and an efficient symbolic algorithm for solving a general backward tridiagonal linear systems of the form (1.1). To do this we begin by translate the system (1.1) to the following tridiagonal linear system:

$$A_1 X = Y_1, \tag{2.1}$$

where

$$A_1 = \begin{bmatrix} d_n & b_n & 0 & \cdots & 0 & 0 & 0 \\ a_{n-1} & d_{n-1} & b_{n-1} & \cdots & 0 & 0 & 0 \\ 0 & a_{n-2} & d_{n-2} & b_{n-2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_2 & d_2 & b_2 \\ 0 & 0 & 0 & \cdots & 0 & a_1 & d_1 \end{bmatrix}, \tag{2.2}$$

$Y_1 = (y_n, y_{n-1}, \ldots, y_1)^{\mathrm{T}}$ and $n \geqslant 3$.

Now considering the LU decomposition [3] of the matrix A_1 in the form:

$$\begin{bmatrix} d_n & b_n & 0 & \cdots & 0 & 0 & 0 \\ a_{n-1} & d_{n-1} & b_{n-1} & \cdots & 0 & 0 & 0 \\ 0 & a_{n-2} & d_{n-2} & b_{n-2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_2 & d_2 & b_2 \\ 0 & 0 & 0 & \cdots & 0 & a_1 & d_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{a_{n-1}}{\beta_1} & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{a_{n-2}}{\beta_2} & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{a_2}{\beta_{n-2}} & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{a_1}{\beta_{n-1}} & 1 \end{bmatrix} \begin{bmatrix} \beta_1 & b_n & 0 & \cdots & 0 & 0 & 0 \\ 0 & \beta_2 & b_{n-1} & \cdots & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & b_{n-2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \beta_{n-1} & b_2 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \beta_n \end{bmatrix}. \tag{2.3}$$