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Multiple-soliton solutions for the Lax–Kadomtsev–Petviashvili (Lax–KP) equation

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Abstract

A new completely integrable dispersive equation is derived. The new equation is obtained by extending the Lax fifth-order equation using the sense of the Kadomtsev–Petviashvili (KP) equation in extending the KdV equation. The newly derived Lax–Kadomtsev–Petviashvili (Lax–KP) equation is investigated by using the tanh–coth method and the Hirota bilinear method to derive single soliton solution and N-soliton solutions, respectively. The study highlights the multiple-soliton solutions of the derived completely integrable equation.

Keywords: Hirota bilinear method; Hereman's method; tanh–coth Method; Lax equation; KP equation; Multiple-soliton solutions

1. Introduction

It is well-known that multi-soliton solutions of completely integrable evolution equations can be obtained by three different methods; the inverse scattering method, the Bäcklund transformation method, and the Hirota bilinear method [1–5]. Each of these methods is widely used and has its own features. However, the Hirota's bilinear method is rather heuristic and gives multiple soliton solutions for a wide class of nonlinear evolution equations in a direct method. The Hereman's simplified form of the Hirota's bilinear method facilitates the computation work to derive N-soliton solutions.

The aim of this work is twofold. First, we aim to derive a new completely integrable dispersive equation. We fellow the Lax [6] approach by generalizing the bilinear form of the KdV equation to obtain the well-known Lax fifth-order nonlinear equation. We next follow the Kadomtsev–Petviashvili sense [7] to extend the Lax equation to the new completely integrable Lax–Kadomtsev–Petviashvili (Lax–KP) equation. The next goal is to apply the tanh–coth method [8–16] to determine single soliton and trigonometric solutions. We further aim to use a combination of Hirota's method [1–5,17–20] and Hereman's simplified form [21,22] to determine multiple-soliton solutions [23–26].

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2. The Lax–Kadomtsev–Petviashvili equation

In this section, we will derive the Lax–Kadomtsev–Petviashvili equation (Lax–KP). It is well-known that the KdV equation

$$u_t + 6uu_x + u_{xxx} = 0, \tag{1}$$

can be expressed in terms of the bilinear operators [1,17]

$$D_x(D_t + D_x^3)f \cdot f = 0. \tag{2}$$

On the other hand, the KP equation extends the KdV equation, and is given by

$$(u_t + 6uu_x + u_{xxx})_x + u_{yy} = 0, \tag{3}$$

that can be expressed in terms of the bilinear operators as

$$[D_x(D_t + D_x^3) + D_y^2]f \cdot f = 0. \tag{4}$$

where the Hirota's bilinear operators [1] are defined by the following rule

$$D_t^n D_x^m a \cdot b = \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m a(x,t)b(x',t')|x' = x, t' = t. \tag{5}$$

The solution of Eq. (1) is of the form

$$u(x,t) = 2\frac{\partial^2 \ln f(x,t)}{\partial x^2}. \tag{6}$$

Lax [6] generalized (2) and (6) into the form

$$\begin{aligned} &[D_x(D_t + D_x^5) - \frac{5}{3}D_s(D_s + D_x^3)]f \cdot f = 0, u(x,t) = 2\frac{\partial^2 \ln f(x,t)}{\partial x^2}, \\ &D_x(D_s + D_x^3)f \cdot f = 0, \end{aligned} \tag{7}$$

by involving an auxiliary variable s. Eq. (7) can be written as

$$\begin{aligned} &D_xD_t(f \cdot f) + D_x^6(f \cdot f) - \frac{5}{3}D_s^2 - \frac{5}{3}D_sD_x^3 = 0, u(x,t) = 2\frac{\partial^2 \ln f(x,t)}{\partial x^2}, \\ &D_xD_sf, f + D_x^4 f \cdot f = 0. \end{aligned} \tag{8}$$

The Hirota's bilinear operators have several properties [1,17] such as

$$\begin{aligned} &D_xD_t(f \cdot f) = f^2(\ln f^2)_{xt}, \\ &D_x^6(f \cdot f) = f^2(u_{4x} + 15uu_{2x} + 15u^3) \end{aligned} \tag{9}$$

Substituting (9) into (8) gives

$$2(\ln f)_{xt} + 10u^3 + 10uu_{2x} + 5u_x^2 + u_{4x} = 0. \tag{10}$$

Differentiate (10) with respect to x and using (6) gives the Lax fifth-order equation

$$u_t + 30u^2u_x + 20u_xu_{2x} + 10uu_{3x} + u_{5x} = 0. \tag{11}$$

Following the sense of the KP Eq. (3) we can extend the Lax equation to the Lax–Kadomtsev–Petviashvili (Lax–KP) in the form

$$[u_t + 30u^2u_x + 20u_xu_{2x} + 10uu_{3x} + u_{5x}]_x + u_{yy} = 0. \tag{12}$$

The Lax–KP equation is a completely integrable evolution equation that has an infinite conservation laws of energy and hence multiple-soliton solutions. This equation will be examined by using the tanh–coth method [8–16] for single soliton solution, and by using the Hirota's bilinear form for multiple-soliton solutions.