Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 201 (2008) 587–596
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

k-Restricted edge connectivity for some interconnection networks ☆

Shiying Wang [a,*], Jun Yuan [a], Aixia Liu [b]

[a] School of Mathematical Sciences, Shanxi University, Taiyuan, Shanxi 030006, People's Republic of China
[b] Business College of Shanxi University, Taiyuan, Shanxi 030031, People's Republic of China

Abstract

k-Restricted edge connectivity is an important parameter in measuring the reliability and fault tolerance of large interconnection networks. In this paper we present two families of graphs similar with the networks proposed by Chen et al. [Y.C. Chen, J.J.M. Tan, L.H. Hsu, S.S. Kao, Super-connectivity and super edge-connectivity for some interconnection networks, Applied Mathematics and Computation 140 (2003) 245–254] and study the k $(k = 2, 3)$-restricted edge connectivity of these graphs. In particular, as the applications of our results, the k $(k = 2, 3)$-restricted edge connectivity of the recursive circulant graphs and the n-ary k-cubes is given.

Keywords: Edge connectivity; Restricted edge connectivity; k-Restricted edge connectivity

1. Introduction

It is well known that when the underlying topology of an interconnection network is modeled by a connected graph $G = (V, E)$, where V is the set of processors and E is the set of communication links in the network, the edge connectivity $\lambda(G)$ of G is an important measurement for reliability and fault tolerance of the network [14]. In general, the larger $\lambda(G)$ is, the more reliable the network is. The parameter, however, has an obvious deficiency, that is, it tacitly assume that all edges incident with the same vertex of G can potentially fail at the same time, which is almost impossible in the practical applications of networks. In other words, in the definition of $\lambda(G)$, absolutely no restrictions are imposed on the components of $G - S$ where S is an edge set such that $G - S$ is disconnected. Consequently, the measurement is inaccurate for large-scale processing systems in which all links incident with the same processor cannot fail at the same time. To compensate for this shortcoming, it would seem natural to generalize the notion of the classical edge connectivity by imposing some restrictions on the components of $G - S$. Fàbrega and Fiol [7] introduced the k-restricted edge connectivity of interconnection networks.

☆ This work is supported by the National Natural Science Foundation of China (10471081).
* Corresponding author.
E-mail address: shiying@sxu.edu.cn (S. Wang).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.12.055

For graph–theoretical terminology and notation not defined here we follow [2]. We consider finite, undirected and simple graph G with the vertex set $V(G)$ and the edge set $E(G)$. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v and the edge neighborhood $NE_G(v)$ of v in G to be the set of edges incident with v. The degree $d_G(v)$ of a vertex v is the number of vertices in $N_G(v)$. The girth of G is the length of a shortest cycle in G. For a vertex set $X \subset V$, denote the induced subgraph of X in G by $G[X]$. For two vertex sets X, Y of G, we define $[X, Y] = \{uv : u \in X, v \in Y \text{ and } uv \in E(G)\}$. A graph G is k-regular if $d_G(v) = k$ for every vertex v in G. An edge cut is a set $S \subseteq E(G)$ such that $G - S$ is disconnected. The edge connectivity $\lambda(G)$ of G is the minimum size of an edge cut. G is maximally edge-connected if $\lambda(G) = \delta(G)$, where $\delta(G)$ is the minimum degree of G. For neighborhoods, edge neighborhoods and degrees, we will usually omit the subscript for the graph when it is clear which graph is meant.

Definition 1.1. An edge set $S \subseteq E(G)$ is a k-restricted edge cut if $G - S$ is disconnected and every component of $G - S$ has at least k vertices. The k-restricted edge connectivity of G, denoted by $\lambda_k(G)$, is defined as the cardinality of a minimum k-restricted edge cut.

In particular, the 2-restricted edge cut is called the restricted edge cut and the 2-restricted edge connectivity is called the restricted edge connectivity. We denote the restricted edge connectivity of G by $\lambda'(G)$. In view of previous studies on k-restricted edge connectivity, it seems that the larger $\lambda_k(G)$ is, the more reliable the network is [9,13]. So we expect $\lambda_k(G)$ to be as large as possible. Clearly, the optimization of $\lambda_k(G)$ requires an upper bound first. For any positive integer k, let $\xi_k(G) = \min\{|[X, \overline{X}]| : |X| = k \text{ and } G[X] \text{ is connected}\}$. It has been shown that $\lambda_k(G) \leqslant \xi_k(G)$ for many graphs [3,10,17].

Two families of interconnection networks proposed by Chen et al. [4] are presented as follows.

Definition 1.2. Let G_0 and G_1 be two disjoint graphs with the same number of vertices. The graph $G(G_0, G_1; M)$ is such a graph with vertex set $V(G_0) \cup V(G_1)$ and edge set $E(G_0) \cup E(G_1) \cup M$, where M is a perfect matching between the vertices of G_0 and G_1; i.e., a set of $|V(G_0)|$ nonadjacent edges with one endpoint in G_0 and the other endpoint in G_1.

Definition 1.3. Let $G_0, G_1, \ldots, G_{r-1}$ be disjoint graphs with $|V(G_i)| = n$ for $i = 0, 1, \ldots, r-1$. The graph $H = G(G_0, G_1, \ldots, G_{r-1}; \mathscr{M})$ is such a graph with $V(H) = V(G_0) \cup V(G_1) \cup \cdots \cup V(G_{r-1})$ and $E(H) = \mathscr{M} \cup \bigcup_{i=0}^{r-1} E(G_i)$ and $\mathscr{M} = \bigcup_{i=0}^{r-1} M_{i,i+1(\bmod r)}$, where $M_{i,i+1(\bmod r)}$ is an arbitrary perfect matching between $V(G_i)$ and $V(G_{i+1(\bmod r)})$.

In this paper, we present two families of interconnection networks as follows.

Definition 1.4. Let t, n be two positive integers with $t \leqslant n$ and let G_0 and G_1 be two disjoint graphs with $V(G_0) = \{a_0, \ldots, a_{n-1}\}$ and $V(G_1) = \{b_0, \ldots, b_{n-1}\}$. The graph $G(G_0, G_1; M_t)$ is such a graph with vertex set $V(G_0) \cup V(G_1)$ and edge set $E(G_0) \cup E(G_1) \cup M_t$, where $M_t = \{a_i b_j : j - i (\bmod n) \leqslant t - 1 \text{ and } i, j = 0, 1, \ldots, n-1\}$.

Definition 1.5. Let r, t, n be three positive integers with $r \geqslant 3, t \leqslant n$ and let $G_0, G_1, \ldots, G_{r-1}$ be disjoint graphs with $V(G_i) = \{a_{i0}, a_{i1}, \ldots, a_{i(n-1)}\}$ for $i = 0, 1, \ldots, r-1$. The graph $H = G(G_0, G_1, \ldots, G_{r-1}; \mathscr{M}_t)$ is such a graph with $V(H) = V(G_0) \cup V(G_1) \cup \cdots \cup V(G_{r-1})$ and $E(H) = \mathscr{M}_t \cup \bigcup_{i=0}^{r-1} E(G_i)$ and $\mathscr{M}_t = \bigcup_{i=0}^{r-1} M_{i,i+1(\bmod r)}$, where $M_{i,i+1(\bmod r)} = \{a_{ij} a_{i+1(\bmod r)j'} : j' - j (\bmod n) \leqslant t - 1 \text{ and } j, j' \in \{0, 1, \ldots, n-1\}\}$ for $i \in \{0, 1, \ldots, r-1\}$.

In some sense, $G(G_0, G_1; M_1) = G(G_0, G_1; M)$. So $G(G_0, G_1; M_t)$ generalizes the interconnection network $G(G_0, G_1; M)$. $G(G_0, G_1, \ldots, G_{r-1}; \mathscr{M}_t)$ is an interesting interconnection network related with $G(G_0, G_1; M_t)$ and $G(G_0, G_1, \ldots, G_{r-1}; \mathscr{M})$.

Chen and Tan [5] discussed the restricted edge connectivity of $G(G_0, G_1; M)$ and $G(G_0, G_1, \ldots, G_{r-1}; \mathscr{M})$ and showed the following results.

Theorem 1.6 [5]. *Let k and n be two positive integers with $n > k + 1$. Let G_0 and G_1 be two disjoint k-regular graphs, $|V(G_0)| = |V(G_1)| = n$, containing no triangle. Assume that $\lambda(G_0) = \lambda(G_1) = k$. Then, $G = G(G_0, G_1; M)$ is $(k+1)$-regular, containing no triangle, $\lambda(G) = k + 1$, and $\lambda'(G) = 2k$.*