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# Dynamics of a predator-prey system with pulses

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ABSTRACT

In this paper, we investigate the dynamic behaviors of a Holling II two-prey one-predator system with impulsive effect concerning biological control and chemical control strategy-periodic releasing natural enemies and spraying pesticide at different fixed moment. By using the Floquet theory of linear periodic impulsive equation and small-amplitude perturbation method, we show that there exists a globally asymptotically stable two-prey eradication periodic solution when the impulsive period is less than some critical value. Further, we prove that the system is permanent if the impulsive period is larger than some critical value, and meanwhile the conditions for the extinction of one of the two-prey and permanence of the remaining two species are given. Finally, we give numerical simulation, with increasing of predation rate for the super competitor and impulsive period, the system displays complicated behaviors including a sequence of direct and inverse cascades of periodic-doubling, periodic-halving, chaos and symmetry breaking bifurcation. Our results suggest a new approach in the pest control.



## 1. Introduction

It is well known, many insects are beneficial to human, but some insects are harmful to human, only these harmful insects can cause economic damage as their population reaching economic injury level. Controlling harmful insects and other arthropods has become an important issue in recent years, for instance, minimizing losses due to insect pests and insect vectors remains an essential component of the programmes in the Office of Agricultural Entomology in China. Overusing a single control tactic is discouraged to minimize damage to non-target organisms, and to preserve the quality of the environment. Overusing a single control tactic also can lead pest to produce resistance to chemical control (for example, pesticide), it will be more difficult to control pest later. Then biological and chemical control were introduced.

Biological control [1–7] is the purposeful introduction one or more natural enemies of an exotic pest, specifically for the purpose of suppressing the abundance of the pest in a new target region to a level at which it no longer causes economic damage. Virtually all insect and mite pests have some natural enemies. Natural enemies are able to play a more active role in suppressing insect pests. Usually, predators feed on not only insect pests but also other insects. There may be more than one pest species – for example, the two species of aphids predominant in small grains: the English grain aphid and the oat-bird cherry aphid. Aphids' high reproductive rate enables their populations to quickly build up to levels that can cause an economic loss. However, aphids are usually kept in check by biological control agents, such as lady beetles, parasitic wasps, and syrphid fly maggots which are often abundant in small grains. One approach to biological control is augmentation, which is manipulation of existing natural enemies to increase their effectiveness. This can be achieved by mass production and periodic releasing natural enemies, and by genetic enhancement of the enemies to increase their effectiveness at control.

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Wherever possible, different pest control techniques should work together rather than against each other. In this paper, according to periodic biological and chemical control, and based on two-prey one-predator system with Holling II functional response, we suggest a simple mathematical model with pulses to describe the process of periodic releasing natural enemies and spraying pesticide (or harvesting pests) at different fixed moments. System with impulsive effects describing evolution processes are characterized by the fact that at certain moments of time they abruptly experience a change of state. Processes of such character are studied in almost every domain of applied sciences. Numerous examples are given in Bainov's and his collaborator's books [8,9]. Some impulsive equations have been recently introduced into population dynamics in relation to: vaccination [10,11], population ecology [12,13], and impulsive birth [14,15], chemotherapeutic [16,17].

The paper is arranged like this. A Holling II two-prey one-predator system concerning biological and chemical control is given in Section 2. In Section 3, we give some notations and lemmas. In Section 4, we consider the local stability and global asymptotic stability of the two-pest eradication periodic solution by using Floquet theory for the impulsive equation, small-amplitude perturbation skills and techniques of comparison, and in Section 5 we show that the system is permanent if the impulsive period is larger than some critical value. Moreover, we give the sufficient conditions for one of two-prey extinction and the remaining two species permanence. A brief discussion and further numerical simulation are given in the last section.

## 2. Model formulation

Based on many experiments, Holling [18] suggested three different kinds of functional response for different kinds of species to model the phenomenon of predation, which made the standard Lotka–Volterra systems more realistic. Liu and Chen [13] investigated complex dynamics of Holling type II Lotka–Volterra predator-prey system with impulsive perturbations on the predator. Zhang and Chen [19] studied a Holling II functional response food chain model with impulsive perturbations. The model we considered in this paper is based on the following predator-prey model, where two-prey are competitive and the predator has Holling II functional response, that is

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big(b_1 - x_1(t) - \alpha x_2(t) - \frac{\eta z(t)}{1+\omega_1 x_1(t)}\Big), \\ \dot{x}_2(t) = x_2(t)\Big(b_2 - \beta x_1(t) - x_2(t) - \frac{\mu z(t)}{1+\omega_2 x_2(t)}\Big), \\ \dot{z}(t) = z(t)\Big(-b_3 + \frac{d\eta x_1(t)}{1+\omega_1 x_1(t)} + \frac{d\mu x_2(t)}{1+\omega_2 x_2(t)}\Big), \end{cases} \tag{2.1}$$

where $x_i(t)$ $(i = 1, 2)$ is the population size of prey (pest) species and $z(t)$ is the population size of predator (natural enemies) species, $b_i > 0$ $(i = 1, 2, 3)$ are intrinsic rates of increase or decrease, $\alpha > 0$ and $\beta > 0$ are parameters representing competitive effects between two-prey, $\eta > 0$ and $\mu > 0$, $\frac{\eta x_1(t)z(t)}{1+\omega_1 x_1(t)}$ and $\frac{\mu x_2(t)z(t)}{1+\omega_2 x_2(t)}$ are the Holling type II functional responses, $d > 0$ is the rate of conversing prey into predator.

Model (2.1) with constant periodic releasing predator and spraying pesticide (or harvesting pests) was studied by Song and Li [20]:

$$\begin{cases} \left.\begin{aligned} &\dot{x}_1(t) = x_1(t)\Big(b_1 - x_1(t) - \alpha x_2(t) - \tfrac{\eta z(t)}{1+\omega_1 x_1(t)}\Big), \\ &\dot{x}_2(t) = x_2(t)\Big(b_2 - x_2(t) - \beta x_1(t) - \tfrac{\mu z(t)}{1+\omega_2 x_2(t)}\Big), \\ &\dot{z}(t) = z(t)\Big(-b_3 + \tfrac{d\eta x_1(t)}{1+\omega_1 x_1(t)} + \tfrac{d\mu x_2(t)}{1+\omega_2 x_2(t)}\Big), \end{aligned}\right\} \quad t \neq nT, \\ \left.\begin{aligned} &\Delta x_1(t) = -p_1 x_1(t), \\ &\Delta x_2(t) = -p_2 x_2(t), \\ &\Delta z(t) = q, \end{aligned}\right\} \quad t = nT, \end{cases} \tag{2.2}$$

where $\Delta x_i(t) = x_i(t^+) - x_i(t), \Delta z(t) = z(t^+) - z(t)$. $T$ is the period of the impulsive effect, $p_i > 0 (i = 1, 2)$ is the proportionality constant which represents the rate of mortality due to applying pesticide. $q > 0$ is the number of predator released each time.

Now we will develop systems (2.1) and (2.2) by introducing a constant periodic releasing natural enemies and spraying pesticide (or harvesting pests) at different fixed moment. That is, we consider the following impulsive differential equations:

$$\begin{cases} \left.\begin{aligned} &\dot{x}_1(t) = x_1(t)\Big(b_1 - x_1(t) - \alpha x_2(t) - \tfrac{\eta z(t)}{1+\omega_1 x_1(t)}\Big), \\ &\dot{x}_2(t) = x_2(t)\Big(b_2 - x_2(t) - \beta x_1(t) - \tfrac{\mu z(t)}{1+\omega_2 x_2(t)}\Big), \\ &\dot{z}(t) = z(t)\Big(-b_3 + \tfrac{d\eta x_1(t)}{1+\omega_1 x_1(t)} + \tfrac{d\mu x_2(t)}{1+\omega_2 x_2(t)}\Big), \end{aligned}\right\} \quad t \neq (n+l-1)T, t \neq nT, \\ \left.\begin{aligned} &\Delta x_i(t) = -p_i x_i(t), \\ &\Delta z(t) = -pz(t), \end{aligned}\right\} \quad t = (n+l-1)T, \\ \left.\begin{aligned} &\Delta x_i(t) = 0, \\ &\Delta z(t) = q, \end{aligned}\right\} \quad t = nT, \end{cases} \tag{2.3}$$