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A globally and superlinearly convergent smoothing Broyden-like method for solving nonlinear complementarity problem

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Abstract

The nonlinear complementarity problem (denoted by NCP(F)) has attracted much attention due to its various applications in economics, engineering and management science. In this paper, we propose a smoothing Broyden-like method for solving nonlinear complementarity problem. The algorithm considered here is based on the smooth approximation Fischer–Burmeister function and makes use of the derivative-free line search rule of Li in [D.H. Li, M. Fukushima, A derivative-free line search and global convergence of Broyden-like method for nonlinear equations, Optim. Meth. Software 13(3) (2000) 181–201]. We show that, under suitable conditions, the iterates generated by the proposed method converge to a solution of the nonlinear complementarity problem globally and superlinearly.

Keywords: Nonlinear complementarity problem; Smoothing Broyden-like method; Global convergence; Superlinear convergence

1. Introduction and algorithm

The nonlinear complementarity problem [2–4] is to find a vector $x \in R^n$ such that

$$x \geqslant 0, \quad F(x) \geqslant 0, \quad x^{\mathrm{T}}F(x) = 0. \tag{1.1}$$

where $F: R^n \to R^n$ is a given function. Throughout this paper, we assume that F is continuously differentiable P_0-function.

It is well knows that NCP(F) is equivalent to system of equations in the form of

$$\Phi(x) = 0, \tag{1.2}$$

where $\Phi: R^n \to R^n$ is a semismooth function. Such a function can be obtained, for example, by Fischer–Burmeister function:

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$$\phi_{FB}(a,b) = a + b - \sqrt{a^2 + b^2}$$

and

$$\Phi(x) = \begin{pmatrix} \phi_{FB}(x_1, F_1(x)) \\ \vdots \\ \phi_{FB}(x_n, F_n(x)) \end{pmatrix}. \tag{1.3}$$

It is easy to show that $\phi_{FB}(a,b) = 0$ holds if and only if $a \geqslant 0$, $b \geqslant 0$ and $ab = 0$. Fischer–Burmeister function is differentiable at every point except the origin point and is semismooth at the origin point. By introducing a smoothing parameter, we obtain a smoothing Fischer–Burmeister function

$$\phi(\mu, a, b) = a + b - \sqrt{a^2 + b^2 + \mu^2}, \tag{1.4}$$

where μ is a nonnegative parameter. It is clear that $\phi(0, a, b) = \phi_{FB}(a,b)$.

Next, we recall some useful definitions and results.

Definition 1.1

(1) A matrix $M \in R^n$ is said to be a P_0-matrix if all its principal minors are nonnegative.
(2) A function $F: R^n \to R^n$ is said to be a P_0-function if for all $x, y \in R^n$ with $x \neq y$, there exists an index $i_0 \in N$ such that

$$x_{i_0} \neq y_{i_0}, \quad (x_{i_0} - y_{i_0})[F_{i_0}(x) - F_{i_0}(y)] \geqslant 0.$$

Lemma 1.1. *Let $\mu > 0$ and the function $\phi: R_{++} \times R^2$ be defined by (1.4). Let $\{a_k\}$, $\{b_k\}$ be any two sequences such that $a_k, b_k \to +\infty$ or $a_k \to -\infty$ or $b_k \to -\infty$. Then For any $(\mu, a, b) \in R_{++} \times R^2$, we have $|\phi(\mu, a_k, b_k)| \to +\infty$.*

Proof. The proof can be founded in Ref. [5]. □

Let $z = (\mu, x) \in R_{++} \times R^n$ and

$$H(z) = H(\mu, x) = \begin{pmatrix} \mu \\ \Phi(z) \end{pmatrix}, \tag{1.5}$$

where

$$\Phi(z) := \Phi(\mu, x) = \begin{pmatrix} \phi(\mu, x_1, F_1(x)) \\ \vdots \\ \phi(\mu, x_n, F_n(x)) \end{pmatrix}. \tag{1.6}$$

Thus, NCP(F) (1.1) is equivalent to the following equation:

$$H(z) = 0 \tag{1.7}$$

in the sense that their solution sets are coincident.

By simple calculation, it is not difficult to see that $H(\cdot)$ is continuously differentiable at any $z = (\mu, x) \in R_{++} \times R^n$ with its Jacobian

$$H'(z) = \begin{pmatrix} 1 & 0 \\ v(z) & D_1(z) + D_2(z)F'(x) \end{pmatrix}, \tag{1.8}$$

where

$$v(z) := \text{vec}\{v_i(z) = \phi'_\mu(\mu, x_i, F_i(x)) : i \in N\},$$
$$D_1(z) := \text{diag}\{a_1(z), a_2(z), \ldots, a_n(z)\},$$
$$D_2(z) := \text{diag}\{b_1(z), b_2(z), \ldots, b_n(z)\}$$