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New explicit and exact solutions for a system of variant RLW equations

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Abstract

In this article, a system of regularized long wave equations are studied. With the aid of the mathematic software Maple and using the direct method, some new exact solutions: compactons, solitons, solitary patterns and periodic solutions are obtained.

Keywords: $R(m,n)$ equations; Compacton; Soliton; Solitary pattern solution; Periodic solution

1. Introduction

The investigation of the exact solutions of nonlinear partial differential equations (PDEs) play an important role in the study of nonlinear physical phenomena. For example, the wave phenomena observed in fluid dynamics, plasma and elastic media are often modelled by the bell-shaped sech functions and the kink-shaped tanh functions. The exact solution, if available, of those nonlinear PDEs facilitates the verification of numerical solvers and aids in the stability analysis of solutions. In the past several decades, various methods for obtaining exact solutions of nonlinear PDEs have been presented, such as inverse scattering method [1], Darboux transformation method [2,3], Hirota bilinear method [4], Lie group method [5,6], bifurcation method of dynamic systems [7], sine–cosine method [8–13], tanh function method [12–14], Fan-expansion method [15,16], homogenous balance method [17] and so on.

Recently, Dye and Parker [18] studied the well-known nonlinear regularized long wave equation (RLW equation)

$$u_t + au_x - 6uu_x + bu_{xxt} = 0 \tag{1}$$

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by using inverse scattering method. This RLW equation was introduced as an alternative model to the KdV equation to describe small amplitude long wave in shallow water. An increasing interest in studying the equation has been attached many mathematicians to investigate the problem to develop new solutions and to examine the physical behavior of the obtained solutions. More recently, Wazwaz [12] introduced a system of nonlinear variant RLW equations

$$u_t + au_x - k(u^n)_x + b(u^n)_{xxt} = 0 \tag{2}$$

and derived some compact and noncompact exact solutions by using the sine–cosine method and the tanh method. Motivated by the rich mathematical and physical properties of the RLW Eqs. (1) and (2), in this paper we study the following generalized variant RLW equations ($R(m,n)$ equations in short):

$$R(m,n) : u_t + au_x - k(u^m)_x + b(u^n)_{xxt} = 0. \tag{3}$$

With the aid of Maple, we obtain some new exact solutions such as compactons, solitary pattern solutions, solitons and periodic solutions.

2. Explicit and exact solutions of $R(m,n)$ equations

Now we seek the travelling wave solutions of (3). Let

$$u(x,t) = u(\xi), \quad \xi = x - ct, \tag{4}$$

where c is wave speed. Substituting (4) into (3) yields

$$(a-c)u_\xi - k(u^m)_\xi - bc(u^n)_{\xi\xi\xi} = 0. \tag{5}$$

Integrating (5) once, we obtain

$$(a-c)u - ku^m - bc(u^n)_{\xi\xi} - g = 0, \tag{6}$$

where g is an integral constant.

Below we seek compacton solutions and solitary pattern solutions of (3) using the four ansatzs

$$\text{Ansatz 1:} \quad u(x,t) = \begin{cases} A\cos^\delta(B\xi), & |B\xi| \leqslant \frac{\pi}{2}, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

$$\text{Ansatz 2:} \quad u(x,t) = \begin{cases} A\sin^\delta(B\xi), & |B\xi| \leqslant \pi, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

$$\text{Ansatz 3:} \quad u(x,t) = A\ \cosh^\delta(B\xi). \tag{9}$$

$$\text{Ansatz 4:} \quad u(x,t) = A\ \sinh^\delta(B\xi), \tag{10}$$

where A, B, δ are parameters to be determined later.

2.1. Compact and noncompact solutions of ansatz 1

Substituting (7) into (3) gives

$$A(a-c)\cos^\delta(B\xi) - A^m k\cos^{m\delta}(B\xi) + bcn^2\delta^2A^nB^2\cos^{n\delta}(B\xi) - bcn\delta A^nB^2(n\delta-1)\cos^{n\delta-2}(B\xi) - g = 0. \tag{11}$$

Thus we can obtain the three possible cases to be discussed:

Case 1. When $n\delta - 2 = 0, \delta = m\delta = n\delta$, we have

$$\begin{cases} A(a-c) - A^m k + bcn^2\delta^2A^nB^2 = 0, \\ bcn\delta A^nB^2(n\delta-1) + g = 0 \end{cases} \tag{12}$$