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A note on the compound binomial model with randomized dividend strategy

Zhen-hua Bao

School of Mathematics, Liaoning Normal University, Dalian 116029, China

Abstract

This paper considers the compound binomial model with randomized decisions on paying dividends. By using two formulas obtained by Tan and Yang [J.Y. Tan, X.Q. Yang, The compound binomial model with randomized decisions on paying dividends, Insurance: Mathematics and Economics 39 (2006) 1–18], two defective renewal equations for the Gerber–Shiu penalty function are derived and solved. The analytic solutions obtained are utilized to derive the probability of ultimate ruin, the deficit distribution at ruin and the distribution of the claim causing ruin. The asymptotic estimate satisfied by the penalty function is discussed in some detail.

Keywords: Compound binomial model; Defective renewal equation; Dividend strategy; Gerber–Shiu penalty function; Asymptotic estimate

1. Introduction

The classical compound binomial risk model is a discrete time risk process with the following features. The premium received in each period is 1. In any period the probability of claim is p $(0 < p < 1)$, and probability of no claim is $q = 1 - p$. We assume that claims occur at the end of the period and denote by $\xi_t = 1$ the event where a claim occurs in period $(t-1, t]$ and $\xi_t = 0$ the event where no claim occurs in the time period $(t-1, t]$. The claims are independent and identically distributed (i.i.d.) positive integer valued random variables distributed as a random variable X with cumulative distribution function (c.d.f.) $\overline{F}(x) = 1 - F(x)$ and probability function (p.f.) $f(x),\ x \in N_+$. Where we denote by N the set of nature numbers and $N_+ = N - \{0\}$. Throughout, we assume the equilibrium p.f. of X is $f_1(x) = \frac{\overline{F}(x-1)}{E(X)},\ x \in N_+$, and $F_1(x) = 1 - \overline{F}_1(x)$ is its c.d.f. For $t = 0, 1, \ldots$, the surplus at time t is

$$U(t) = u + t - \sum_{i=1}^{t} X_i \xi_i, \tag{1}$$

E-mail address: zhhbao@126.com

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where $U(0) = u$, which is a non-negative integer. The risk model (1) has been studied extensively since it was introduced by Gerber [6]. Gerber derived a formula for the ruin probability, the distribution of the deficit at ruin and the distribution of the surplus just before ruin in the case that the initial surplus is 0. Shiu [14] derived similar expressions as Gerber's using different methods. Cheng et al. [2] derived the moment generating function of the time to ruin for zero initial surplus and then they derived recursively the joint distribution of the surplus prior to ruin and the deficit at ruin. Dickson [4] applied elementary methods to derive the joint distribution of the surplus before ruin and the deficit at ruin when the initial reserve is 0.

Recently, Gerber and Shiu [7] introduced a discounted penalty function with respect to the time of ruin, the surplus immediately before ruin and the deficit at ruin, which has proven to be a powerful analytical tool. See also Lin and Willmot [11]. For the compound binomial model, Pavlova and Willmot [13] derived a defective renewal equation satisfied by the Gerber–Shiu discounted penalty function.

Dividend strategies for insurance risk models were first proposed by De Finetti [3] to reflect more realistically the surplus cash flows in an insurance portfolio. Barrier strategies for the classical compound Poisson risk model have been studied extensively in the literature, including Albrecher et al. [1], Dickson and Waters [5], Gerber and Shiu [7,8], Lin et al. [12], Lin and Pavlova [10], among many others.

In Tan and Yang [15], the authors considered the compound binomial model modified by the inclusion of randomized dividend strategies. The insurer will pay a dividend of 1 with a probability q_0 $(0 \leqslant q_0 < 1)$ in each time period if the surplus is greater than or equal to a non-negative integer b at the beginning of the period. That is, for $t = 0, 1, 2, \ldots$, the surplus process at time t is given by

$$U(t) = u + t - \sum_{k=1}^{t} \eta_k I(U(k-1) \geqslant b) - \sum_{k=1}^{t} X_k \xi_k, \tag{2}$$

where $I(E)$ is the indicator function of an event E, η_k $(k \geqslant 1)$ is a series of randomized decision functions which are i.i.d. and independent of $\left\{\sum_{k=1}^{t} X_k \xi_k\right\}$. In detail, we denote by $\eta_k = 1$ the event where a dividend of 1 is paid at the time k and $\eta_k = 0$ the event when no dividend is paid at the time k. We assume

$$P(\eta_k = 1) = q_0; \qquad P(\eta_k = 0) = p_0,$$

where $p_0 + q_0 = 1$.

Note that if $q_0 = 0$, the dividend risk model reduces to the classical model without constraints, see Tan and Yang [15] for more motivations about this dividend strategy. In which the authors obtained the recursive formula and asymptotic estimate for the Gerber–Shiu discounted free penalty function.

We also assume that the positive security loading condition holds. That is, if we denote by θ the relative security loading then

$$\theta = \frac{1 - p\mu - q_0}{p\mu} > 0, \tag{3}$$

where $\mu = E(X) < \infty$.

We now introduce the Gerber–Shiu discounted penalty function. Let $T = \{t \in N_+; U(t) < 0\}$ be the time of ruin in the modified model (2) with $T = \infty$ if ruin does not occur. Note that if ruin occurs, $|U(T)|$ is the deficit at ruin and $U(T-)$ is the surplus immediately prior to ruin. Denote by

$$m_v(u) = E[v^T \omega(U(T-), |U(T)|) I(T < \infty) | U(0) = u], \tag{4}$$

the Gerber–Shiu expected discounted penalty function. Here, $\omega(u_1, u_2) : N \times N_+ \to N$ is a non-negative bounded function, $0 < v \leqslant 1$ is the discount factor. We remark that there is a slight difference from Tan and Yang's in the definition (compare Eq. (4) with Eq. (2.11) in Tan and Yang [15]).

The aim of this paper is to study the Gerber–Shiu discounted free penalty function for risk model (2). For simplicity, we write $m(u) = m_1(u)$. The rest of the paper is organized as follows: In Section 2, we derive two defective renewal equations satisfied by the penalty function. Analytical solutions of the two renewal equations are presented in Section 3. As applications, we apply the analytical solutions to the probability of ultimate ruin, the deficit at ruin $|U(T)|$ and the claim causing ruin $U(T-) + |U(T)|$. In Section 4, we derive the asymptotic estimation for the penalty function, which is simpler and more computable than Tan and Yang's.