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Multiple positive periodic solutions for a class of state-dependent delay functional differential equations with feedback control

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Abstract

By employing Krasnoselskii fixed point theorem, we investigate the existence of multiple positive periodic solutions for a class of state-dependent delay functional differential equations with feedback control. The system considered in this paper is more general and incorporates as special cases various problems which have been studied extensively in the literature. Moreover, some easily verifiable sufficient criteria are established.

Keywords: State-dependent delay; Positive periodic solution; Functional differential equations; The Krasnoselskii fixed point theorem; Feedback control

1. Introduction

Let $\mathbf{R} = (-\infty, +\infty)$, $\mathbf{R}_+ = [0, +\infty)$, $\mathbf{R}_- = (-\infty, 0]$ and $\mathbf{R}^\mathbf{n}_+ = \{(x_1, \ldots, x_n)^\mathrm{T} : x_i \geqslant 0, 1 \leqslant i \leqslant n\}$, respectively. For each $x = (x_1, x_2, \ldots, x_n)^\mathrm{T} \in \mathbf{R}^n$, the norm of x is defined as $|x|_0 = \sum_{i=1}^n |x_i|$. Let **BC** denote the Banach space of bounded continuous functions $\phi : \mathbf{R} \to \mathbf{R}^\mathbf{n}$ with the norm $\|\phi\| = \sup_{\theta \in R} \sum_{i=1}^n |\phi_i(\theta)|$, where $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^\mathrm{T}$.

Recent years have witnessed increasing interest in ecosystem with feedback controls [2,3,5–9,11,12]. The reasons for introducing control variables are based on main two points. On one hand, ecosystem in the real world are continuously distributed by unpredictable forces which can results in changes in the biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables (for more details, one can see [6]). On the other hand, in the literature, it has been proved that, under certain conditions, some species are permanence but some are possible extinction in the competitive system, for example, see [1]. In order to search

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for certain schemes to ensure all the species coexist, feedback control variables should be introduced to ecosystem.

Compared with advanced in the area of studying the existence of a unique periodic solution [3–5,7–9,12], less progress has been achieved in studying the existence of multiple periodic solutions, only several papers concern this subject, see [13–15]. It is worth mentioning that one of effective approaches to fulfill such a problem is employing fixed point theorem, and some prior estimations of possible periodic solutions are obtained. In the present paper, by utilizing the fixed point theorem due to Krasnoselskii, we aim to study the existence of multiple periodic solutions. Using the work of [8] as our starting point, we proceed to develop more results in state-dependent delay functional equations with feedback controls, which is formulated as follows:

$$\begin{cases} \dot{x}(t) = A(t,x(t))x(t) + f(t,x_t,x(t-\tau(t,x(t))),u(t-\alpha(t))), \\ \dot{u}(t) = B(t,x(t))u(t) + C(t,x(t))x(h(t,x(t))), \end{cases} \tag{1.1}$$

where $A(t, x(t)) = \mathrm{diag}[a_1(t, x(t)), \ldots, a_n(t, x(t))]$, $B(t, x(t)) = \mathrm{diag}[b_1(t, x(t)), \ldots, b_n(t, x(t))]$, $C(t, x(t)) = \mathrm{diag}[c_1(t, x(t)), \ldots, c_n(t, x(t))]$, a_i, b_i, $c_i \in C(R\times R, R)$,$\alpha \in C(R, R)$ are ω-periodic, $\tau(t, y)$,$h(t, y) \in C(R\times R^n, R)$ satisfy $\tau(t+\omega, y) = \tau(t, y)$, $h(t+\omega, y) = h(t, y)$ for all $t \in R$, $y \in R^n$. $f = (f_1, f_2, \ldots, f_n)^{\mathrm{T}}$, $f(t, x_t, y, z)$ is a function defined on $R\times BC\times R^n\times R^n$ and $f(t+\omega, x_{t+\omega}, y, z) = f(t, x_t, y, z)$ whenever x is ω-periodic. If $x \in BC$, then $x_t \in BC$ for any $t \in R$, where x_t is defined by $x_t(\theta) = x(t+\theta)$ for $\theta \in R$.

To conclude this section, we summarize in the following related definition and the famous fixed point theorem that will be needed in our arguments.

Definition. Let X be Banach space and E be a closed, nonempty subset of X, E is said to be a cone if

(i) $\alpha u + \beta v \in E$ for all u, $v \in E$ and all α, $\beta > 0$,
(ii) u, $-u \in E$ imply $u = 0$.

Lemma 1.1 (Krasnoselskii fixed point theorem [10]). *Let X be a Banach space, and let E be a cone in X. Suppose Ω_1 and Ω_2 are open subsets of X such that $0 \in \Omega_1 \subset \bar{\Omega}_1 \subset \Omega_2$. Suppose that*

$$T : E \cap (\overline{\Omega}_2 \setminus \Omega_1) \to E$$

is a completely continuous operator and satisfies either

(i) $\|Tx\| \geqslant \|x\|$ *for any* $x \in E\cap\partial\Omega_1$ *and* $\|Tx\| \leqslant \|x\|$ *for any* $x \in E\cap\partial\Omega_2$; *or*
(ii) $\|Tx\| \leqslant \|x\|$ *for any* $x \in E\cap\partial\Omega_1$ *and* $\|Tx\| \geqslant \|x\|$ *for any* $x \in E\cap\partial\Omega_2$.

Then T has a fixed point in $E \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

2. Preliminaries

In this section, we make some prepare for next section.

Lemma 2.1. $(x(t), u(t))^{\mathrm{T}}$ *is an ω-periodic solution of (1.1) if and only if it is also an ω-periodic solution of the following system*:

$$\begin{aligned} &\dot{x}(t) = A(t,x(t))x(t) + f(t,x_t,x(t-\tau(t,x(t))),u(t-\alpha(t))), \\ &u(t) = \int_t^{t+\omega} \overline{G}(t,s)C(s,x(s))x(h(s,x(s)))\mathrm{d}s := (\Phi x)(t) \end{aligned} \tag{2.1}$$

where

$$\overline{G}(t,s) = \mathrm{diag}[\overline{G}_1(t,s), \overline{G}_2(t,s), \ldots, \overline{G}_n(t,s)],$$

and

$$\overline{G}_i(t,s) = \frac{\exp\{-\int_t^s b_i(r,x(r))\mathrm{d}r\}}{\exp\{-\int_0^\omega b_i(r,x(r)\mathrm{d}r\} - 1}, \quad s \in [t, t+\omega], \quad 1 \leqslant i \leqslant n. \tag{2.2}$$