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Optimization by k-Lucas numbers

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Abstract

This article presents a mathematical analysis of Fibonacci search method by k-Lucas numbers. In this study, we develop a new algorithm which determines the maximum point of unimodal functions on closed intervals. As a result, it makes Fibonacci search method more effective.

Keywords: Optimization; Fibonacci numbers; k-Lucas numbers

1. Introduction

It is well known that the Fibonacci sequence has various applications. The optimization problem is one of them. By using Fibonacci search algorithms, maximum or minimum point of a given unimodal function on a closed interval $[a,b]$ can be determined. Before proceeding further, let us introduce the Fibonacci sequence $\{F_n\}$. The sequence $\{F_n\}$ which is defined as $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ is called Fibonacci sequence. This sequence can be written as

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

A generalization of the Fibonacci sequence is called the k-Fibonacci sequence for positive integers $k \geqslant 2$. The k-Fibonacci sequence $\{g_n^{(k)}\}$ is defined as

$$g_1^{(k)} = \cdots = g_{(k-2)}^k = 0, \quad g_{(k-1)}^k = g_{(k)}^k = 1$$

and for $n > k \geqslant 2$,

$$g_n^{(k)} = g_{n-1}^{(k)} + g_{n-2}^{(k)} + \cdots + g_{n-k}^{(k)}.$$

$g_n^{(k)}$ is called the nth k-Fibonacci number. Under the definition above, the k-Fibonacci sequence [2] can be written as follows:

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$$g_{k+1}^{(k)} = g_k^{(k)} + g_{k-1}^{(k)} = 1 + 1 = 2,$$
$$g_{k+2}^{(k)} = g_{k+1}^{(k)} + g_k^{(k)} + g_{k-1}^{(k)} = 2 + 1 + 1 = 2^2,$$
$$\dots$$
$$\dots$$
$$\dots$$
$$g_{2k-2}^{(k)} = g_{2k-3}^{(k)} + \dots + g_k^{(k)} + g_{k-1}^{(k)} = 2^{k-3} + \dots + 2 + 1 + 1 = 2^{k-2},$$
$$g_{2k-1}^{(k)} = g_{2k-2}^{(k)} + \dots + g_k^{(k)} + g_{k-1}^{(k)} = 2^{k-2} + 2^{k-3} + \dots + 2 + 1 + 1 = 2^{k-1},$$

which implies that $g_j^k = 2^{j-k}$ for $j = k, k+1, \dots, 2k-1$. For instance, if $k = 2$, then $\{g_n^{(2)}\}$ is the 2-Fibonacci sequence.

The k-Lucas sequence $\{L_n^{(k)}\}$ is a sequence which is defined as $L_n^{(k)}=f_{n-1}^{(k)}+f_{n+k-1}^{(k)}$. Each term in this sequence is called k-Lucas number. It is well known that $L_j^{(k)} = 2^{j-1}$, $1 \leqslant j \leqslant k-1$ and $L_k^{(k)} = 1 + 2^{k-1}$.

As mentioned above Fibonacci and Lucas numbers have many applications. Lucas numbers are used in the problems for which the maximum number of evaluation needed to reduce the interval of uncertainty i.e., by using k-Lucas numbers we reduce the interval of uncertainty to within prescribed length faster [3]. We can find the maximum point of a given unimodal function on a closed interval $[a, b]$ by evaluating the end points of the reduced intervals. They are determined according to k-Lucas numbers.

In this paper, we develop a new algorithm which determine the maximum point of a given unimodal function by using k-Lucas numbers. The algorithm in [1] which determines the minimum point of a unimodal function is not enough to find the minimum point of any unimodal function. In this paper, we develop an algorithm which finds the maximum point of any given unimodal function.

Based on the algorithm in [1], a new algorithm which finds the maximum point of a given unimodal function, is written. However we examine that this algorithm does not work for any unimodal function. We determine the lack of the algorithm in [1] and develop a new algorithm which gives better results and work for any unimodal function.

2. Modified Fibonacci search algorithm

Let us consider the following algorithm which determines the maximum point of any given unimodal function $f(x)$

$$f(x)_{x\in[a,b]} \to \max$$

Let l_m^k denote mth k-Lucas number. The algorithm, we develop, is written as follows:

Step 1. Input a, b, $f(x)$
Step 2. for $m = 1$ to n
Step 3. Choose as $[a_1,b_1] = [a,b]$ and $m = 1$
Step 4. If $f(a_m) < f(b_m)$ then $y_{\max}=f(b_m)$ otherwise $y_{\max}=f(a_m)$
Step 5. Calculate the points

$$x_m = a_m + (b_m - a_m)\frac{l_{n-m+1}^k}{l_{n-m+3}^k}, \quad x'_m = a_m + (b_m - a_m)\frac{l_{n-m+2}^k}{l_{n-m+3}^k}$$

Step 6. Find the values $f(x_m)$ and $f(x'_m)$
Step 7.
Case I. $f(x_m) \leqslant f(x'_m)$ and $y_{\max}=f(b_m)$ then $[a_{m+1},b_{m+1}] = [x_m, b_m]$
Case II. $f(x_m) \leqslant f(x'_m)$ and $y_{\max}=f(a_m)$
if $f(x'_m) \leqslant y_{\max}$ then $[a_{m+1}, b_{m+1}] = [a_m, x'_m]$
otherwise $[a_{m+1},b_{m+1}] = [x_m,b_m]$
Case III. $f(x_m) \geqslant f(x'_m)$ and $y_{\max}=f(a_m)$ then $[a_{m+1}, b_{m+1}] = [a_m, x'_m]$