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Periodicity in a nonlinear predator–prey system on time scales with state-dependent delays ☆

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Abstract

With the help of a continuation theorem based on Gaines and Mawhin's coincidence degree, easily verifiable criteria are established for the global existence of positive periodic solutions of a nonlinear state-dependent delays predator–prey system on time scales.

Keywords: Time scales; Periodic solutions; Nonlinear; State-dependent delay; Predator–prey system; Coincidence degree

1. Introduction

It is well known that a very basic and important problem in the study of a population model with a periodic environment is the global existence and stability of positive periodic solution. Much progress has been made in this direction, (see, for example [8–12,14,18–23,30,31] and the references cited therein). We see that many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different from their continuous counterparts. Careful investigation reveals that it is similar to explore the existence of periodic solutions for nonautonomous population models governed by ordinary differential equations and their discrete analogue in the approaches, the methods and the main results. For example, extensive research reveals that many results concerning the existence of periodic solution of predator–prey systems modelled by differential equations can be carried over to their discrete analogues based on the coincidence theory, for example [21–25,27]. Recently, Bohner et al. [5] pointed out that it is unnecessary to explore the existence of periodic solutions of some continuous and discrete population models in separate ways. One can unify such studies in the sense of dynamic equation on general time scales.

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On the other hand, recently, several scholars [10–14,21,33] gave their attention to nonlinear population dynamics, since such systems could simulate the real world more accurately. Recently, Chen [11] has studied following system:

$$\begin{cases} \frac{\mathrm{d}N_1(t)}{\mathrm{d}t} = N_1(t)\left[b_1(t) - \sum_{i=1}^{n} a_i(t)(N_1(t-\tau_i(t,N_1(t),N_2(t))))^{\alpha_i}\right. \\ \qquad \left. - \sum_{j=1}^{m} c_j(t)(N_2(t-\sigma_j(t,N_1(t),N_2(t))))^{\beta_j}\right], \\ \frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = N_2(t)\left[-b_2(t) + \sum_{i=1}^{n} d_i(t)(N_1(t-\rho_i(t,N_1(t),N_2(t))))^{\gamma_i}\right], \end{cases} \tag{1.1}$$

where $a_i(t)$, $c_j(t)$, $d_i(t)$ are continuous positive periodic functions with periodic $\omega > 0$, $b_1(t)$, $b_2(t)$ are continuous periodic functions with period ω and $\int_0^{\omega} b_i(t)\mathrm{d}t > 0$. τ_i, σ_j, ρ_i $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$ are continuous and ω-periodic with respect to their first arguments, respectively. α_i, β_j, γ_i $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$ are positive constants.

Chen [12] has investigated following nonlinear discrete state-dependent delay predator–prey model:

$$\begin{cases} N_1(k+1) = N_1(k)\exp\left[b_1(k) - \sum_{i=1}^{n} a_i(k)(N_1(k-\tau_i(k,N_1(k),N_2(k))))^{\alpha_i}\right. \\ \qquad \left. - \sum_{j=1}^{m} c_j(k)(N_2(k-\sigma_j(k,N_1(k),N_2(k))))^{\beta_j}\right], \\ N_2(k+1) = N_2(k)\exp\left[-b_2(k) + \sum_{i=1}^{n} d_i(k)(N_1(k-\rho_i(k,N_1(k),N_2(k))))^{\gamma_i}\right], \end{cases} \tag{1.2}$$

where a_i, c_j, d_i: $Z \to R^+$ are positive ω-periodic, ω is a fixed positive integer. b_1, b_2: $Z \to R^+$ are ω-periodic and $\sum_{k=0}^{\omega-1} b_i(k) > 0$. τ_i, σ_j, ρ_i: $Z \times R \times R \to R$ $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$ are ω-periodic with respect to their first arguments, respectively. α_i, β_j, γ_i $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$ are positive constants.

It is natural to ask that whether we can also unify the studies of periodic solution of such nonlinear population models with state-dependent delays?

The theory of measure chains, which has recently received a lot of attention, see [1–7,15–17,32], was introduced by Hilger in his Ph.D. thesis [26] in 1988 in order to unify continuous and discrete analysis. Though there has been much research activities concerning the oscillation and nonoscillation of solutions of differential equation on time scales (or measure chains) (see, for example [2,15–17,29,32]), there are few results dealing with the periodic solutions of nonlinear predator–prey systems with state-dependent delay system.

Motivated by the above works, we consider the following system on time scales:

$$\begin{cases} x_1^{\Delta}(t) = b_1(t) - \sum_{i=1}^{n} a_i(t)\exp\left\{\alpha_i x_1(t-\tau_i(t,e^{x_1(t)},e^{x_2(t)}))\right\} \\ \qquad - \sum_{j=1}^{m} c_j(t)\exp\left\{\beta_j x_2(t-\sigma_j(t,e^{x_1(t)},e^{x_2(t)}))\right\}, \\ x_2^{\Delta}(t) = -b_2(t) + \sum_{i=1}^{n} d_i(t)\exp\left\{\gamma_i x_1(t-\rho_i(t,e^{x_1(t)},e^{x_2(t)}))\right\}, \end{cases} \tag{1.3}$$