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An evolution strategy method for computing eigenvalue bounds of interval matrices

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Abstract

In this paper, a modified method by means of an evolution strategy (ES) is introduced for computing eigenvalue bounds of interval matrices. Then a sufficient condition theorem of the evolutionary strategy is presented which guarantees the convergence in probability of the method. The numerical examples show the method can effectively yield accurate bounds for interval eigenvalues.

Keywords: Evolution strategy (ES); Interval matrix; Eigenvalue bounds; Convergence in probability

1. Introduction

The eigenvalue problems with interval matrices dealt with in this paper can be formulated as

$$Ku = \lambda u \tag{1}$$

subject to

$$\underline{K} \leqslant K \leqslant \overline{K} \quad \text{or} \quad \underline{k_{ij}} \leqslant k_{ij} \leqslant \overline{k_{ij}}, \quad \text{for } i,j = 1,2,\ldots,n, \tag{2}$$

where $\underline{K} = (\underline{k_{ij}})$, $\overline{K} = (\overline{k_{ij}})$ and $K = (k_{ij})$ are all $n \times n$ real symmetric matrices. $\underline{K}$ and $\overline{K}$ are known matrices which compose of the lower and upper bounds of the intervals respectively. K is an uncertain-but-bounded matrix and ranges over the inequalities in (2). λ is the eigenvalue of the eigenvalue problem (1) with unknown matrix K, and u is the corresponding eigenvector of λ.

By using the interval matrix notations [1,2], inequalities (2) can be written as $K \in K^I$ in which $K^I = [\underline{K}, \overline{K}]$ is a symmetric interval matrix. So the problem is: for a given interval matrix K^I, find an eigenvalue interval λ^I, that is,

$$\lambda^I = [\underline{\lambda}, \overline{\lambda}] = (\lambda_i^I), \quad \lambda_i^I = [\underline{\lambda_i}, \overline{\lambda_i}] \tag{3}$$

such that it encloses all possible eigenvalues λ satisfying $Ku = \lambda u$, when $K \in K^I$.

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The problem formulated by (1) and (2) arises in many practical mechanic and engineering fields, as the structural parameters are often uncertain because of the inaccurate measurements, errors in manufacture, changes of nature environment, etc. In the case of uncertain quantities considered to be unknown, except that they belong to given sets in a parameter space, all information about structural systems may be summarized by interval matrices, interval vectors or intervals [1,3,4]. It is often necessary to estimate bounds for eigenvalues of some complex systems with these uncertain parameters. Eq. (1) which describe the uncertainty is called interval description.

Many studies on this aspect have been done for the problems occurring in such as engineering and control fields in the past decades. Since the mid-sixties, the interval analysis [5] has been proposed, which has become a useful tool in many fields. Moore [1], Alefeld and Herzberger [6] have carried out the pioneering work. Mori and Kokame [7] and Argoun [8] discussed the sufficient and necessary conditions of the dynamic stability for the uncertain systems. Sobld et al. [9], and Rachid [10] discussed the robustness of control systems with uncertain parameters. Qiu et al. [3] developed the rayleigh quotient iteration method for solving interval eigenvalues in structures with interval parameters. Yang et al. [11] proposed a method based on the interval finite element method and evaluated bounds of complex eigenvalues for damping systems with interval parameters.

To our knowledge, there exist mainly two kinds of methods in computing eigenvalue bounds for the interval eigenvalue problem (1). The first kind of methods is based on perturbation theory. Under the condition that the signs of the components of eigenvectors remain invariable, Deif developed an effective method [4] for the standard interval eigenvalue problem by using the eigenvalue inequalities and nonlinear programming theorems. This method can yield good bounds. Furthermore, as a consequence, under Deif's conditions, Dimarogonas [12] advanced that the eigenvalue bounds should be reached just as the matrix elements took their bound values. However, there exists no criterion for judging the conditions, so the application of Deif's method is restricted. Qiu et al. presented a method in [13] which based on an interval perturbation approximating formula. This method moved away certain limitations for matrices which required by Deif's method. However, it can only produce *approximate* but not true bounds for eigenvalues. That is, there exist some matrices belonging to the matrix interval, such that the exact eigenvalues of which can exceed the bounds obtained by this method. Also [14] presented a new method based on perturbation theory to compute eigenvalue bounds of standard interval eigenvalue problems. This method needs no any preconditions and takes less computational work than Deif's method, but the bounds obtained are a little worse than Deif's.

The second kind of methods, which studied by fewer researchers, is based on certain optimization theory. In the methods, the interval eigenvalue problem is considered to be a constrained optimization problem and the exact eigenvalue bounds can be obtained if the optimization arithmetic had no defection. In [15], a direct optimization method of SQP (sequential quadratic programming) was proposed. But some current arithmetics for constrained nonlinear optimization problems were based on the theories of extreme values. It follows that the computed results are sometimes local optimal solutions so that the methods of [15] cannot guarantee that the computed results are global optimal solutions.

To overcome the drawback of the direct optimization method, in the following, a modified method using evolution strategies is proposed for computing eigenvalue bounds of the problem (1). Numerical examples are presented to show the method is reliable and effective. A sufficient condition of convergence of ES is ALSO discussed.

2. The ES method for computing eigenvalue bounds

2.1. Introduction of ESs

Evolution strategies (ESs) are algorithms which imitate the principle of natural evolution as a method to solve parameter optimization problems [16]. Evolution strategy methods can be understood as 'intelligent' probabilistic search algorithms which are based on the evolutionary process of biological organisms in nature. The ESs were originally developed by Rechenberg [17] and Schwefel [18]. During the course of evolution, natural populations evolve according to the principles of natural selection and 'survival of the fittest'. Individuals which are more successful in adapting to their environment will have a better chance of surviving and reproducing, whilst individuals which are less fit will be eliminated. This means that the highly fit individuals