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Goal programming in the context of the assignment problem and a computationally effective solution method

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Abstract

This paper develops the goal programming technique to solve the multiple objective assignment problem. The required model is formulated and an appropriate solution method is presented. The proposed method, which is a decomposition method, exploits the total unimodularity feature of the assignment problem and effectively reduces the computational efforts. Some issues related to the efficiency of a GP solution are stated and some specialized techniques for detecting and restoring efficiency are proposed.

Keywords: Goal programming; Assignment problem; Multi-objective programming; Pareto efficiency; Dantzig–Wolfe reformulation

1. Introduction

The assignment problem (AP) is a 0–1 programming problem that arises in a variety of applications. This special case of the transportation problem has attracted a great deal of attention in the literature. The classical assignment problem involves assigning m individuals to m jobs. There is a cost c_{ij} for each partial assignment of individual i to job j. The following model is the classical assignment problem that finds an assignment with the minimum cost.

$$\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{m} x_{ij} = 1, \quad i = 1,\ldots,m, \\
& \sum_{i=1}^{m} x_{ij} = 1, \quad j = 1,\ldots,m, \\
& x_{ij} = 0 \text{ or } 1, \quad i,j = 1,\ldots,m.
\end{aligned}$$

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There are a number of variants for the assignment problem such as the multi-objective AP [1, p. 253] and the generalized AP [2]. In this paper, goal programming (GP) in the context of the assignment problem is formulated and investigated. Goal programming is a strategy for solving multi-objective problems in which a set of goals is wished to be achieved. Goal programming that was implicitly proposed by Charnes et al. [3] and then stated more explicitly by Charnes and Cooper [4] is amongst most practical multi-objective strategies [5]. Two major forms of GP are weighted and lexicographic GP. The general formulation of weighted GP is

$$\begin{aligned}\text{Min}\quad & z=\sum_{k=1}^{t}(u_k n_k + v_k p_k) \\ \text{s.t.}\quad & f_k(\mathbf{x}) + n_k - p_k = b_k, \quad k=1,\ldots,t, \\ & \mathbf{x}\in C_s, \\ & n_k, p_k \geqslant 0, \quad k=1,\ldots,t, \\ & \mathbf{x} \geqslant \mathbf{0},\end{aligned} \tag{1}$$

where C_s is a set of hard constraints, f_k is the kth objective, and b_k is the goal value for the kth objective. n_k and p_k are the negative and positive deviations from the goal b_k, respectively. $u_k n_k + v_k p_k$ is called the achievement function for the kth objective (u_k and v_k are non-negative weights). If either u_k or v_k is zero, the corresponding achievement function is called one-sided, otherwise it is called two-sided. In this paper, a deviational variable is called undesirable if it is included in an achievement function, otherwise it is called desirable.

In the lexicographic GP, the objective function of program (1) is replaced with

$$\text{Lex Min}\, a = \left[\sum_{k=1}^{t}(u_k^1 n_k + v_k^1 p_k), \ldots, \sum_{k=1}^{t}(u_k^L n_k + v_k^L p_k)\right], \tag{2}$$

and optimization is done in order of their importance. In other words, the objectives with lower priorities are taken into account just after those in higher priorities.

Unfortunately, GP in the assignment problem environment is a 0–1 program. This paper proposes a computationally effective algorithm based on the Dantzig–Wolfe decomposition method [6–8] to solve this program. This algorithm exploits the special structure of the assignment problem as far as possible.

A criticism of GP is the possibility of producing inefficient solutions. A feasible solution is inefficient if at least one of its objectives can be improved without worsening the others. A number of methods have been proposed to overcome this deficiency (see for example [9] for the continuous case and [10] for the integer case). Some of these methods are for detecting inefficient objectives and some of them are for restoring the efficiency of the GP solution. The appropriate adjustments for detecting and restoring the efficiency of the resulted GP solution in the assignment problem environment will be proposed in this paper. It will be also shown that the above-mentioned decomposition algorithm can be used here in a similar way.

Section 2 presents the formulation of the GP model in the framework of the assignment problem and proposes a decomposition-based method to solve it. In Section 3, efficiency detection and restoration techniques are adjusted to the under-discussion GP problem. A numerical example for the proposed decomposition algorithm is described in Section 4.

2. Goal programming in the context of the assignment problem

If the set of hard constraints in model (1) is replaced with the constraints set in AP and the assignment aggregated costs are used as the objective functions in model (1), the following weighted sum GP model is obtained:

$$\begin{aligned}\text{Min}\quad & z=\sum_{k=1}^{t}(u_k n_k + v_k p_k) \\ \text{s.t.}\quad & \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}^k x_{ij} + n_k - p_k = b_k, \quad k=1,\ldots,t,\end{aligned} \tag{3}$$