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On goodness-of-fit tests for multiple recursive random number generators

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Abstract

This paper considers the problem of reducing the computational time in testing uniformity for a full period multiple recursive generator (MRG). If a sequence of random numbers generated by a MRG is divided into even number of segments, say $2s$, then the multinomial goodness-of-fit tests and the empirical distribution function goodness-of-fit tests calculated from the ith segment are the same as those of the $(s+i)$th segment. The equivalence properties of the goodness-of-fit test statistics for a MRG and its associated reverse and additive inverse MRGs are also discussed.

Keywords: Goodness-of-fit; Monte Carlo; Multiple recursive generator; Random number

1. Introduction

The generation of an ideal sequence of independent and uniform random numbers (RNs) is of importance in Monte Carlo simulation. Multiple recursive generator (MRG), generalized feedback shift register generator, inverse generator, and combined generator are four approaches to generate a sequence of RNs. For a good survey and detailed discussion of these four RN generators, we refer the reader to see [1–4]. In this paper, we concentrate on a MRG, defined as

$$X_n \equiv a_1X_{n-1} + a_2X_{n-2} + \cdots + a_kX_{n-k} (\bmod m) \quad \text{for } n > k, \tag{1}$$

where m is the modulus usually chosen to be the largest prime number less than the computer's word size, $a_k \neq 0$, $a_j \neq 0$ for at least one j, $1 \leqslant j < k$, and $X_1, X_2, \ldots, X_k$ are starting values such that at least one is nonzero.

Long period, randomness, and efficient implementation are three prerequisites for an ideal RN generator. The more efficient implementation results in placing more restriction on the multipliers [5]. Under some restrictions on the multipliers, various random tests with statistical powerful and computational efficient criteria have

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been proposed to detect departures from randomness in a full period MRG to obtain an ideal one. In the literature, two kinds of statistical tests are distinguished to evaluate the randomness of the set of all vectors of successive RNs generated by a MRG. One is the theoretical test to examine the global randomness behaviors. The other is the empirical test to examine the local randomness behaviors. Among the theoretical tests, the spectral test and lattice test have been widely applied to sequences in order to assess the global randomness behavior. The empirical tests can be classified as either independent tests or uniform tests. In the literature, the adopted statistical tests include the chi-square, Kolmogorov–Smirnov and serial tests for testing uniformity, while runs and auto-correlation tests for testing independence in different dimensions.

According to the statistical powerful and computational efficient criteria, the spectral test is widely suggested as the most powerful tool for assessing the randomness behavior among the statistical tests. In designing ideal MRGs, even if we restrict the full period vector of multipliers $(a_1, a_2, \ldots, a_k)$ with the objective of maximizing the spectral test performance, the rapid computation of a full period MRG with high spectral value is a rather difficult task because of the multiple local maxima of the spectral test. It follows that an optimal vector of multipliers procedure typically relies on an exhaustive search. However, the number of possible vectors of multipliers is extremely large, so an exhaustive search for the kth-order MRG with order $k > 1$ appears to be impractical because of the heavy computational burden. As a result, deducing the equivalence property of periodicity and spectral value of a MRG to reduce the number of possible vectors of multipliers is a subject of considerable ongoing research. For a kth-order MRG (1), Tang [6,7] constructed two associated ones. One associated MRG is the additive inverse of X_n, defined as

$$Y_n^{\mathrm{I}} \equiv -a_1 Y_{n-1}^{\mathrm{I}} + (-1)^2 a_2 Y_{n-2}^{\mathrm{I}} + \cdots + (-1)^k a_k Y_{n-k}^{\mathrm{I}} (\operatorname{mod} m) \quad \text{for } n > k. \tag{2}$$

Tang [6] established the linear relationship $Y_n^{\mathrm{I}} \equiv (-1)^n X_n (\operatorname{mod} m)$ and showed that the spectral values of these two kth-order MRGs (1) and (2) are equal. For the cases that either k is an even integer or $(m-1)$ is a multiple of four, both of them have the same periods. Another associated MRG generates the same sequence but in reverse order, defined as

$$Y_n^{\mathrm{II}} \equiv -a_{k-1} a_k^{-1} Y_{n-1}^{\mathrm{II}} - a_{k-2} a_k^{-1} Y_{n-2}^{\mathrm{II}} - \cdots - a_1 a_k^{-1} Y_{n-k+1}^{\mathrm{II}} + a_k^{-1} Y_{n-k}^{\mathrm{II}} (\operatorname{mod} m) \quad \text{for } n > k. \tag{3}$$

This reverse MRG (3) possesses the same periods and spectral values as the original MRG (1). Tang and Kung [8] also established a linear relationship between X_n and Y_n^{II}.

From the viewpoint of statistics, this paper deduces the equivalence properties of the goodness-of-fit tests for a MRG to reduce the number of possible vectors of multipliers. If a sequence of random numbers generated by a full period MRG is divided into even number of segments, say $2s$, Kao and Tang [9] showed that the chi-square test calculated from the ith segment is the same as that of the $(s+i)$th segment. Important goodness-of-fit tests may be categorized into two classes (1) multinomial goodness-of-fit tests and (2) empirical distribution function goodness-of-fit tests. The multinomial goodness-of-fit tests, based on multinomial distribution, apply to the discrete observations situation with a finite number of categories. While the empirical distribution function goodness-of-fit tests, based on empirical distribution function, apply to the continuous observations situation with a infinite number of categories. In this paper, this equivalence property is generalized to the multinomial goodness-of-fit tests and the empirical distribution function goodness-of-fit tests. The article is organized as follows. Sections 2 and 3 respectively establish the equivalence properties for the multinomial goodness-of-fit tests and the empirical distribution function goodness-of-fit tests. Section 4 discusses the equivalence properties of the multinomial goodness-of-fit tests and the empirical distribution function goodness-of-fit tests for a MRG and its associated MRGs (2) and (3). Finally, we offer a summary of our findings.

2. Multinomial goodness-of-fit tests

The multinomial goodness-of-fit statistic is a measure of how much the observed category counts diverge from the expected category counts. For the one-dimensional case, since the publication of Pearson's paper in 1900, various ways to measure have been proposed in the literature. All of them are embedded in a family of power divergence statistics proposed by Cressie and Read [10]. More precisely, let n be the total number of RNs to be tested. Assume that the entire range of RNs is divided into c adjacent intervals. Let p_i be the prespec-