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Stabilization of parameters perturbation chaotic system via adaptive backstepping technique

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Abstract

The work of Yassen [M.T. Yassen, Chaos control of chaotic dynamical systems using backstepping design, Chaos Soliton Fract. 27 (2006) 537–548] which mainly investigated the stabilization problem for a class of chaotic systems without the parameters perturbation. This paper is concerned with stabilization problem for a class of parameters perturbation chaotic systems via both backstepping design method and adaptive technique. The proposed controllers can guarantee that the parameters perturbation systems will be stabilized at a fixed bounded point. Furthermore, the paper also proposes controllers to stabilize the uncertain chaotic system at equilibrium point with only backstepping design method. Finally, numerical simulations are given to illustrate the effectiveness of the proposed controllers.

Keywords: Parameters perturbation; Backstepping technique; Adaptive technique; Lorenz system; Chen system; Lu system

1. Introduction

Chaos control has been of a broad interest since OGY [1] method was proposed in 1990. After that lots of methods have been proposed such as time delay feedback control (DFC) [2], bang–bang control [3], optimal control [4], intelligent control [5], Adaptive control [6], differential geometric method [7], backstepping control [8,9], variable structure control [10,11].

Backstepping design technique is a systematic design approach for constructing both feedback control laws and associated Lyapunov functions, which has been widely employed in chaos control [8,9] and chaos synchronization [12–14]. Recently, Ge et al. [8] developed a controller based on adaptive backstepping technique to cope with stabilization problem for a class of uncertain chaotic systems, in which the parameters of the system are unknown constants. Yassen [9] proposed controllers to stabilize the Lorenz, Chen and Lu chaotic system via backstepping design technique, in which the parameters of the system are known constants. In some

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real chaotic systems, the parameters are inevitably disturbed by some unknown reasons, such as, temperature variation and voltage fluctuation. In this case, the applications of the above controllers are limited in Ref. [8,9], and it is very important for real chaotic systems to implement the chaos stabilization instead. The presented paper is to provide controllers to deal with the stabilization problem for a class of parameters perturbation chaotic systems: Lorenz, Chen and Lu systems. The impact of the parameters perturbation is obviated by introducing adaptive law. The new proposed controllers can guarantee that the state of parameters perturbation chaotic system will be stabilized at a fixed bounded point. Meanwhile, we also provide controllers to stabilize the state of the uncertain system to steady point without introducing the adaptive laws. Finally, the Numerical simulations are presented to illustrate the validity and perfect of our proposed controllers.

2. Controlling parameter perturbation uncertain Lorenz system

Consider the parameter perturbation Lorenz system as

$$\begin{cases} \dot{x} = a(t)(y - x), \\ \dot{y} = c(t)x - xz - y, \\ \dot{z} = xy - b(t)z + u_1, \end{cases} \tag{1}$$

where $a(t) \in [\underline{a}, \bar{a}]$, $b(t) \in [\underline{b}, \bar{b}]$, $c(t) \in [\underline{c}, \bar{c}]$, x, y, z are the state variables of the system and u_1 is input which is used to stabilize this chaotic system.

Assumption 1. $\underline{a}$, $\underline{b}$ and $\underline{c}$ are positive constants.

Assumption 1 also holds in other two chaotic systems which will be investigated in Sections 4 and 6, namely, Chen system and Lu system.

Our goal is to find out a proper control law u_1 such that the system (1) will be stabilized at a fixed bounded point.

Starting from the first equation of system (1), an estimative stabilizing function $\alpha_1(x)$ has to be designed for the virtual control y in order to make the derivative of $V_1(x) = (1/2)x^2$, namely $\dot{V}_1(x) = -a(t)x^2 + a(t)xy$, negative definite when $\alpha_1(x) = 0$. Define the error variable w_2 as

$$w_2 = y - \alpha_1(x). \tag{2}$$

Study (x, w_2) system (3)

$$\begin{cases} \dot{x} = a(t)(w_2 - x), \\ \dot{w}_2 = c(t)x - xz - w_2. \end{cases} \tag{3}$$

Consider z as a controller in system (3). Assume when it is equal to $\alpha_2(x, w_2)$, it makes system (3) asymptotically stable. Select Lyapunov function $V_2(x, w_2) = V_1(x) + (1/2)w_2^2$. The derivative of V_2 is

$$\dot{V}_2 = -a(t)x^2 - w_2^2 + w_2 x(a(t) + c(t) - z). \tag{4}$$

Let $\alpha_2(x, w_2) = k$. Since $2pq \leqslant \varepsilon_1 p^2 + \varepsilon_1^{-1} q^2$ for $\varepsilon_1 > 0$, we have

$$\dot{V}_2 \leqslant \left(\varepsilon_1^{-1}(a(t) + c(t) - k)^2/4 - a(t)\right)x^2 + (\varepsilon_1 - 1)w_2^2. \tag{5}$$

If the following inequality group (6) hold, then $\dot{V}_2 < 0$

$$\begin{cases} \varepsilon_1^{-1}(a(t) + c(t) - k)^2/4 - a(t) < 0, \\ \varepsilon_1 - 1 < 0. \end{cases} \tag{6}$$

By straightforward manipulation, we can find out the range of k as

$$\begin{cases} \bar{a} + \bar{c} - 2\sqrt{\varepsilon_1 \underline{a}} < k < \underline{a} + \underline{c} + 2\sqrt{\varepsilon_1 \underline{a}}, \\ 0 < \varepsilon_1 < 1. \end{cases} \tag{7}$$