Available online at www.sciencedirect.com

ScienceDirect

Applied Mathematics and Computation 200 (2008) 267–282

APPLIED MATHEMATICS AND COMPUTATION

www.elsevier.com/locate/amc

Global existence and blow-up solutions and blow-up estimates for a non-local quasilinear degenerate parabolic system ☆

Rui Zhang [a,c], Zuodong Yang [a,b,*]

[a] Institute of Mathematics, School of Mathematics and Computer Science, Nanjing Normal University, Jiangsu, Nanjing 210097, China

[b] College of Zhongbei, Nanjing Normal University, Jiangsu, Nanjing 210046, China

[c] Department of Basic Education, Jinling Institute of Technology, Jiangsu, Nanjing 210001, China

Abstract

This paper deals with p-Laplacian systems

$$\begin{cases} u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = a\int_\Omega u^{\alpha_1}(x,t)v^{\beta_1}(x,t)\,\mathrm{d}x, & x\in\Omega,\ t>0,\\ v_t - \operatorname{div}(|\nabla v|^{q-2}\nabla v) = b\int_\Omega u^{\alpha_2}(x,t)v^{\beta_2}(x,t)\,\mathrm{d}x, & x\in\Omega,\ t>0 \end{cases}$$

with null Dirichlet boundary conditions in a smooth bounded domain $\Omega\subset\mathbf{R}^N$, where $p,q>1,\ \alpha_i,\beta_i\geqslant 0,\ i=1,2$, and $a,b>0$ are positive constants. We first get the non-existence result for a related elliptic systems of non-increasing positive solutions. Secondly by using this non-existence result, blow-up estimates for above p-Laplacian systems with the homogeneous Dirichlet boundary value conditions are obtained under $\Omega=B_R=\{x\in\mathbf{R}^N:|x|<R\}(R>0)$. Then under appropriate hypotheses, we establish local theory of the solutions and obtain that the solutions either exists globally or blow-up in finite time.

Keywords: p-Laplacian systems; Non-local source; Global existence; Non-existence; Blow-up

1. Introduction

In this paper, we investigate the following non-local quasilinear degenerate parabolic system in a smooth bounded domain $\Omega\subset\mathbf{R}^N(N\geqslant 1)$:

☆ Project supported by the National Natural Science Foundation of China (Grant No. 10571022). Project supported by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No. 04KJB110062; 06KJB110056).

* Corresponding author. Address: Institute of Mathematics, School of Mathematics and Computer Science, Nanjing Normal University, Jiangsu, Nanjing 210097, China.

E-mail address: zdyang_jin@263.net (Z. Yang).

0096-3003/$ - see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2007.11.012

$$
\begin{aligned}
&u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = a\int_\Omega u^{\alpha_1}v^{\beta_1}\,\mathrm{d}x, \quad (x,t)\in\Omega\times(0,T],\\
&v_t - \operatorname{div}(|\nabla v|^{q-2}\nabla v) = b\int_\Omega u^{\alpha_2}v^{\beta_2}\,\mathrm{d}x, \quad (x,t)\in\Omega\times(0,T],\\
&u(x,t) = v(x,t) = 0, \quad (x,t)\in\partial\Omega\times(0,T],\\
&u(x,t) = u_0(x), \quad v(x,t) = v_0(x), \quad x\in\Omega,
\end{aligned}
\tag{1.1}
$$

where p, $q > 1$, $\alpha_i, \beta_i \geqslant 0$, $i = 1, 2$ and $a, b > 0$ are positive constants. As well as the non-existence of positive solutions of the related elliptic systems,

$$
\begin{aligned}
&-\operatorname{div}(|\nabla u|^{p-2}\nabla u) = a\int_\Omega u^{\alpha_1}v^{\beta_1}\,\mathrm{d}x, \quad x\in\mathbf{R}^N,\\
&-\operatorname{div}(|\nabla v|^{q-2}\nabla v) = b\int_\Omega u^{\alpha_2}v^{\beta_2}\,\mathrm{d}x, \quad x\in\mathbf{R}^N.
\end{aligned}
\tag{1.2}
$$

For $p = q = 2$, (1.1) is a classical reaction–diffusion system of Fujita type. If $p \neq 2, q \neq 2$, (1.1) appears in the non-Newtonian fluid theory [1–3] as well as in the non-linear filtration theory [4]. In the non-Newtonian fluids theory, the pair (p, q) is a characteristic quantity of medium. Media with $(p, q) > (2, 2)$ are called dilatant fluids and those with $(p, q) < (2, 2)$ are called pseudo-plastics. If $(p, q) = (2, 2)$, they are called Newtonian fluids.

In the last three decades, many authors have studied the following degenerate parabolic problem:

$$
\begin{aligned}
&u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(u), \quad (x,t)\in\Omega\times(0,T],\\
&u(x,t) = 0, \quad (x,t)\in\partial\Omega\times(0,T],\\
&u(x,t) = u_0(x), \quad x\in\Omega,
\end{aligned}
$$

under different conditions. In [5,11,14,15,23–26,28,30,35–37], the existence, uniqueness and regularity of solutions were obtained. When $f(u) = -u^q, q > 0$ or $f(u) \equiv 0$ extinction phenomenon of the solution may appear, see [27,29,31,33]. However, if $f(u) = u^q, q > 1$ the solution may blow up in finite time, see [13,21,30,34]. Roughly speaking, the results in [9,13,21,30,34] read: (1) the solution u exists globally if $q < p - 1$; (2) u blows up in finite time if $q > p - 1$ and $u_0(x)$ is sufficiently large.

Li and Xie [11] studied the following equation:

$$
u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = \int_\Omega u^q(x,t)\mathrm{d}x. \tag{1.3}
$$

with null Dirichlet conditions and obtain that the solution either exists globally or blows up in finite time. Under appropriate hypotheses, they have local theory of the solution and obtain that the solution either exists globally or blow-up in finite time.

Li et al. in [6] deal with the following reaction–diffusion system:

$$
\begin{cases}
u_t - \Delta u = \int_\Omega f(v(y,t))\mathrm{d}y, \quad x\in\Omega,\ t>0,\\
v_t - \Delta v = \int_\Omega g(u(y,t))\mathrm{d}y, \quad x\in\Omega,\ t>0
\end{cases}
\tag{1.4}
$$

with initial and boundary conditions. They proved that there exists a unique classical solution and the solution either exists globally or blow-up in finite time. Furthermore, they obtain the blow-up set and asymptotic behavior provided that the solution blows up in finite time.

For p-Laplacian systems, Yang and Lu [17] studied the following equations:

$$
\begin{cases}
u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = u^{\alpha_1}v^{\beta_1},\\
v_t - \operatorname{div}(|\nabla v|^{q-2}\nabla v) = u^{\alpha_2}v^{\beta_2} \quad x\in\Omega,\ t>0
\end{cases}
\tag{1.5}
$$

they derive some estimates near the blow-up point for positive solutions and non-existence of positive solutions of the relate elliptic systems.