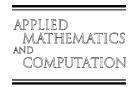




Applied Mathematics and Computation 200 (2008) 308-320



www.elsevier.com/locate/amc

Positive eigenvector of nonlinear perturbations of nonsymmetric M-matrix and Newton iterative solution $\stackrel{\triangleright}{}$

Yao-tang Li*, Shi-liang Wu, Da-ping Liu

Department of Mathematics, Yunnan University, Kunming, Yunnan 650091, PR China

Abstract

Positive eigenvector of nonlinear perturbations of nonsymmetric *M*-matrix and its Newton iterative solution are studied. It is shown that any number greater than the smallest positive eigenvalue of the *M*-matrix is an eigenvalue of the nonlinear problem and that the corresponding positive eigenvector is unique and the Newton iteration of the positive eigenvector is convergent. Moreover, such positive eigenvectors form a monotone increasing and continuous function of the corresponding eigenvalues.

© 2007 Elsevier Inc. All rights reserved.

Keywords: M-Matrix; Nonlinear perturbation; Eigenvalue; Newton iterative solution

1. Introduction

In [1–3], Choi et al. obtained the Perron–Frobenius theorem of nonlinear perturbations of symmetric positive *M*-matrices. The theorem is used to research numerical solutions of Gross–Pitaevskii equation, which plays a important role in modeling of the Bose–Einstein condensation of matter at near absolute zero temperatures, see for example [5], and references therein. However, we know that not all discretization techniques for Gross–Pitaevskii equation lead to a Stieltjes matrix, but an *M*-matrix. In this case, we need a theory on nonlinear perturbations of nonsymmetric *M*-matrices corresponding to the Perron–Frobenius theorem. In this paper, we study this problem and its Newton iterative solution.

We consider the following nonlinear eigenvalue problem:

$$Ax + F(x) = \lambda x,\tag{1}$$

where A is an $n \times n$ nonsingsular M-matrix, the function F(x) is diagonal, that is

E-mail addresses: Liyaotang@ynu.edu.cn (Y.-t. Li), wushiliang1999@126.com (S.-l. Wu).

0096-3003/\$ - see front matter © 2007 Elsevier Inc. All rights reserved. doi:10.1016/j.amc.2007.11.008

[†] This work was done during the visiting at State Key Laboratory of Scientific and Engineering Computing, Chinese Academy of Sciences. I am very grateful to Professor Zhong-ci Shi for his invitation. This work is supported by the Natural Sciences Foundation of Yunnan Province (2005F0011M).

Corresponding author.

$$F(x) = \begin{bmatrix} f_1(x_1) \\ \vdots \\ f_n(x_n) \end{bmatrix}$$

with the property that $f_i(x_i) > 0$ if $x_i > 0$.

Our objectives are to characterize (componentwise) positive solution x of Eq. (1), which are called the positive eigenvector of nonlinear operator G(x) = Ax + F(x), and to research the iterative solution of the positive eigenvector.

Eq. (1) comes from the research of physics, where the Bose–Einstein condensation of atoms at near absolute zero temperatures is modeled by a nonlinear Schrodinger equation in R^3 , also known as the Gross–Pitaevskii equation,

$$-\Delta u + V(x, y, z)u + ku^{3} = \lambda u,$$

$$\lim_{|(x, y, z)| \to \infty} u = 0, \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y, z)^{2} dx dy dz = 1,$$

where V is a given potential function, (e.g. see [6]). The discretization of this equation leads usually to a non-linear eigenvalue problem (1). For example, the discretization of the following one dimensional equation:

$$-u''(x) + c(x)u'(x) + ku^{3}(x) = \lambda u(x), \quad -\infty < x < \infty,$$

$$u(\pm \infty) = 0, \quad \int_{-\infty}^{\infty} u^{2}(x) dx = 1.$$

Using finite difference gives a nonlinear eigenvalue problem $Ax + F(x) = \lambda x$ with matrix

$$A = \frac{1}{h^2} \begin{bmatrix} 2 & -1 + \frac{c_1 h}{2} & 0 & \cdots & 0 \\ -1 + \frac{c_2 h}{2} & 2 & -1 + \frac{c_2 h}{2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 2 \end{bmatrix},$$

and function

$$F(x) = k \begin{bmatrix} x_1^3 \\ x_2^3 \\ \vdots \\ x_n^3 \end{bmatrix},$$

where the c_i 's are the values of c(x) at the mesh point x_i , h is the discretization step-size. When h is small enough, A is an M-matrix. This is a nonlinear eigenvalue problem (1).

Remark 1. Eq. (1) becomes Ax + F(x) = 0 when $\lambda = 0$. Therefore, Ax + F(x) = 0 is a special case of Eq. (1). The theory and solutions of the equation Ax + F(x) = 0 have been studied by several researchers, and its basic theoretical properties, such as existence and uniqueness conditions of solution, and some numerical methods of solving this equation have been obtained, e.g., see [4].

2. Preparatory knowledge

The following lemmas about M-matrix can be found in literatures, see, for example [7–11], we present them here to make the paper self-contained.

Definition 1. An M-matrix A is a nonsingsular real matrix, whose off-diagonal entries are nonpositive, and whose inverse has nonnegative entries only.

Download English Version:

https://daneshyari.com/en/article/4634729

Download Persian Version:

https://daneshyari.com/article/4634729

<u>Daneshyari.com</u>