Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 191 (2007) 302–307
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

A note on the spectral theory of problems on normal oscillation of an ideal compressible fluid in rotating elastic shell

A. Garadzhaev

Department of Applied Mathematics, Turkmen State University, 31 Saparmyrat Turkmenbashy Shayoly, Ashgabat, Turkmenistan

Abstract

In this paper, the structure of the spectrum for certain problems involving normal oscillations of an ideal compressible fluid in rotating elastic shell is investigated. The equivalent system of equations of the problems involving normal oscillations of an ideal compressible fluid in rotating elastic shell is obtained. The case of an elastic container filled with an ideal compressible fluid is studied.

Keywords: Compressible fluids; Fluid–solid interactions; Rotating fluids

1. Introduction

Assume that in the elastic shell Σ occupying an ideal compressible fluid rotating on the fixed axis with small angular velocity $\varepsilon > 0$. We assume that Σ is of class C^3. Let n is a unit exterior normal to Σ which is outside to Ω_0 domain. We will consider Cartesian coordinate system (x_1, x_2, x_3) rotating with the noted mechanic system around the x_3-axis. Let k be a unit vector along this axis. We denote $\omega(x,t)$ as the displacement of the point $x \in \Omega_0$ in the moment t. We will introduce orthogonal coordinate system (α_1, α_2) given on Σ and relative to the lines of curvature of this space.

Let $u(x,t)$ be the coordinate of displacement of the point $x \in \Sigma$ in the body which consist of tangent vectors to the coordinate curves and unit normal vector to Σ. Note that $\rho_1(x,t) = \rho_0 + k_0 p(x,t)$ (**in** Ω_0) is the density of compressible fluid, $\rho(x,t)$ is the density of elastic shell, $p(x,t)$ is the deviation from the balanced pressure of fluid. Putting the solutions

$$u(\alpha_1,\alpha_2,t) = u(\alpha_1,\alpha_2)e^{i\lambda t}, \quad \rho_0\omega(x,t) = \omega(x)e^{i\lambda t}, \quad p(x,t) = p(x)e^{i\lambda t}$$

E-mail address: gamangeldy@yahoo.com

0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.02.087

to the system of free oscillation equation [1, p.77], we get the following eigenvalue problem with respect to λ

$$\frac{eh}{1-\mu^2}Lu - pn - \lambda^2 hpu = 0(\Sigma), \tag{1.1}$$

$$-\frac{1}{k_0}\operatorname{div}\omega = p(\Omega_0), \quad (\omega, n)|_\Sigma = \rho_0(u, n)|_\Sigma, \tag{1.2}$$

$$\lambda^2\omega + 2\varepsilon_i\lambda\omega \times k = \nabla p(\Omega_0). \tag{1.3}$$

Here, e is the elastic modulus, $\mu \in \left(0, \frac{1}{2}\right)$ is the Poisson's coefficient, h is the shell thickness and L is the elliptic operator on Duglas–Nirenberg. Eq. (1.1) has been considered in [2].

In the papers [3,4] the spectral properties of problems on normal oscillations of an ideal incompressible fluid in the rotating elastic containers were investigated. The similar problem with taking into account the fluid compressibility has been considered in [5]. There are also some papers [6,7] dedicated to problems involving small-scale oscillations of an ideal incompressible fluid in nonrotating elastic shells where discovered additional series of points of discrete spectrum appearing with the oscillations of elastic vessel or elastic shell. In the paper [8], the structure of the spectrum for certain problems involving normal oscillations of an ideal compressible fluid in rotating elastic container is investigated.

In the present paper, we investigate the structure of the spectrum for certain problems involving normal oscillations of an ideal compressible fluid in rotating elastic container. We prove that there is a continuous spectrum of the internal waves on $[-2\varepsilon, 2\varepsilon]$ for the case a rotating elastic container filled with an ideal compressible fluid.

2. The main theorem

The problems (1.1)–(1.3) can be reduced to the system of operator equations. We introduce the following spaces:

$$H_0 = \{\varphi \in W_2^1(\Omega_0), (\varphi, 1)_{L_2(\Omega_0)} = 0\}, \quad H_0 \subset W_2^1(\Omega_0),$$
$$H_1 = \{\omega = \nabla\varphi, \varphi \in H_0\}, \quad H_2 = L_2(\Omega_0) \ominus H_1.$$

We get that [4]

$$W_2^1(\Omega_0) \cap H_2 = \{\omega : \operatorname{dir}\omega = 0, (\Omega_0), (\omega, n)|_\Sigma = 0\}. \tag{2.1}$$

Note that in H_0 inner product could be introduced as

$$(\nabla\varphi_1, \nabla\varphi_2)_{L_2(\Omega_0)}$$

and norm generated by this inner product is equivalent to usual norm in the space $W_2^1(\Omega_0)$. As $L_2(\Omega_0) = H_1 \oplus H_2$, we have that for each element $\omega \in L_2(\Omega_0)$ the element $x \in \mathfrak{I} = H_0 \oplus H_2$ by the rule: $x = (\varphi, \vartheta)$, where

$$\omega = \nabla\varphi + \vartheta, \quad \nabla\varphi \in H_1, \quad \vartheta \in H_2 \tag{2.2}$$

at that, if x and y are defined by ω and ω_1 in the accordance with rule then

$$(\omega, \omega_1)_{L_2(\Omega_0)} = (\nabla\varphi, \nabla\varphi_1)_{L_2(\Omega_0)} + (\vartheta, \vartheta_1)_{L_2(\Omega_0)} = (x, y)_\mathfrak{I}$$

that is,

$$(x, y)_\mathfrak{I} = (\omega, \omega_1)_{L_2(\Omega_0)}. \tag{2.3}$$

In other words, indicated one–one mapping between spaces $L_2(\Omega_0)$ and $\mathfrak{I}$ keep the inner product. Now we introduce the bounded linear operator $A : \mathfrak{I} \to \mathfrak{I}, D(A) = \mathfrak{I}$, defined by the formula

$$(Ax, y)_\mathfrak{I} = -\mathrm{i}(\omega x k, \omega_1)_{L_2(\Omega_0)}, \tag{2.4}$$

where ω, ω_1 formed by x, y in the according with the rule (2.3). Now we will transform Eq. (1.3). For that, previously we obtain such element $p_0 \in H_0$, that $\nabla p_0 = \nabla p$. From (1.2) it follows that