Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 191 (2007) 571–581
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Numerical solution of fourth-order problems with separated boundary conditions

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Abstract

In this paper, the fourth-order linear and nonlinear differential equations with separated boundary conditions is converted into an optimal control problem. Then a convergent approximate solution is constructed such that the exact boundary conditions are satisfied. The analysis is accompanied by numerical examples. The obtained results demonstrate reliability and efficiency of the proposed scheme.

Keywords: Least square method; B-splines; Boundary value problems

1. Introduction

Consider the following two-point boundary value problem:

$$\begin{cases} y^{(4)} + f(x)y = g(x), \quad -\infty < a \leqslant x \leqslant b < +\infty, \\ y(a) - A_1 = y(b) - A_2 = y''(a) - B_1 = y''(b) - B_2 = 0, \end{cases} \tag{1}$$

where A_i, B_i, $i = 1, 2$, are finite real constants. The functions $f(x)$ and $g(x)$ are continuous on the interval $[a, b]$. The problem of this type arises in plate deflection theory. Usmani [20] has proved that the above boundary value problem (1) has a unique solution provided

$$\inf_x f(x) = -\eta > -\left[\frac{\pi}{b-a}\right]^4.$$

The deformations of an elastic beam in an equilibrium state, whose two ends are simply supported, can be described by the fourth-order boundary value problems as follows:

$$\begin{cases} y^{(4)} = g(x, y, y''), \quad 0 < x < 1, \\ y(0) = y(1) = y''(0) = y''(1) = 0, \end{cases}$$

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.02.116

where $g : [0,1] \times [0,+\infty) \times (-\infty,0] \to [0,+\infty]$ is continuous [9,10]. Owing to its importance in physics, the existence of solutions to this problem has been studied by many authors. See, for example [3,6,8,11,14,15,21]. But in practice, only its positive solution is significant. In the special case, Ma and Wang [14] studied the existence of positive solutions on the following boundary value problem:

$$\begin{cases} y^{(4)} = f(t,y), & 0 < t < 1, \\ y(0) = y(1) = y''(0) = y''(1) = 0. \end{cases}$$

Li [11] investigated the above generalizing form:

$$\begin{cases} y^{(4)} + \beta y'' - \alpha u = f(t,u), & 0 < t < 1, \\ y(0) = y(1) = y''(0) = y''(1) = 0, \end{cases}$$

under the following conditions:

(c_1) $f(t,y) : [0,1] \times [0,+\infty) \to [0,+\infty)$ is continuous.
(c_2) $\alpha,\beta \in \mathbb{R}$, $\beta < 2\pi^2$, $\alpha \geqslant \frac{-\beta^2}{4}$, $\frac{\alpha}{\pi^4} + \frac{\beta}{\pi^2} < 1$.

Recently, Chai [4] studied the existence of positive solutions on the following boundary value problem:

$$\begin{cases} y^{(4)} + B(t)y'' - A(t)y = f(t,y), & 0 < t < 1, \\ y(0) = y(1) = y''(0) = y''(1) = 0, \end{cases}$$

under the following conditions:

(d_1) $f(t,y) : [0,1] \times [0,+\infty) \to [0,+\infty)$ is continuous.
(d_2) $A(t), B(t) \in C[0,1]$, $\alpha = \inf_{t\in[0,1]} A(t)$, $\beta = \inf_{t\in[0,1]} B(t)$, $\beta < 2\pi^2$, $\alpha \geqslant 0$

$$\frac{\alpha}{\pi^4} + \frac{\beta}{\pi^2} < 1.$$

Usmani [20], Siraj-ul-Islam et al. [18] and Rashidinia et al. [17] used quartic and quantic spline to approximate solution of special fourth-order linear differential equations with two-point boundary conditions (1) by the algorithm of second- or fourth- or sixth-order convergent.

The aim of this paper is to solve the general form of fourth-order differential equations with separated boundary conditions by fourth-degree B-spline functions numerically. These problems have the general form

$$\begin{cases} g(t, y(t), y'(t), y''(t), y'''(t), y^{(4)}(t)) = 0, & -\infty < a \leqslant t \leqslant b < +\infty, \\ \sum_{j=0}^{3} \alpha_{i,j} y^{(j)}(a_i) = A_i, & 1 \leqslant i \leqslant 4, \end{cases} \tag{2}$$

where $a \leqslant a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4 \leqslant b$.

Our presentation finds a sequence of functions $\{v_k\}$ of the form

$$v_k(t) = \sum_{i=-4}^{5\cdot 2^{k-1}-1} c_i B_{ki}(t),$$

which satisfy the exact boundary conditions. Also, up to an error ε_k, the function v_k satisfies the differential equation, where $\varepsilon_k \to 0$ as $k \to \infty$.

2. Explanation of the method

Consider the differential equation

$$g(t, y(t), y'(t), y''(t), y'''(t), y^{(4)}(t)) = 0, \quad a < t < b \tag{3}$$