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A nonlinear programming solution to robust multi-response quality problem

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Abstract

Quite often, engineers obtain measurements associated with several response variables. Both the design and analysis of multi-response experiments with a focus on quality control and improvement have received little attention although they are sorely needed. In a multi-response case the optimization problem is more complex than in the single-response situation. In this paper we present a method to optimize multiple quality characteristics based on the mean square error (MSE) criterion when the data are collected from a combined array. The proposed method will generate more alternative solutions. The string of solutions and the trade-offs aid in determining the underlying *mechanism* of a system or process. The procedure is illustrated with an example, using the generalized reduced gradient (GRG) algorithm for nonlinear programming.

Keywords: Multi-response process optimization; Mean square error; Robust parameter design; Response surface methodology; Nonlinear programming

1. Introduction

One of the conventional approaches used in off-line quality control is the response surface methodology (RSM). However, most previous applications based on RSM have only dealt with a single-response problem and multi-response problems have received only limited attention. In today's complex manufacturing processes call for simultaneous optimization of several quality characteristics rather than optimizing one response at a time. Studies have shown that the optimal factor settings for one performance characteristic are not necessarily compatible with those of other performance characteristics. In more general situations we might consider finding compromising conditions on the input variables that are somewhat favorable to all responses.

There have been some creative methods discussed in the literature for treating multi-response problems. Lind et al. [10] provided an early analysis by overlaying the contour plots of each respected response. This overlay is then examined to identify set of conditions (sometimes called the "sweet spot") that comes as close as possible satisfying all response requirements. In practice, the graphical approach is limited for problems

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where only a few factors and responses are involved. Therefore, there is considerable interest in more general approaches.

The most popular approach to simultaneous response optimization is the desirability function approach, as proposed by Derringer and Suich [3]. The method assigns a scale-free "score" value between zero and one to a response based on the particular goal for that response. A desirability of zero indicates that the predicted value is completely undesirable in terms of the specific response. A desirability of one indicates that the predicted value is completely desirable. The desirabilities (i.e., "score" values) are then combined into a single objective measure to be maximized using a geometric mean function. Köksoy [5] shows how to use the desirability functions of Derringer and Suich to find compromising solutions in a framework for robust design in the case of multiple responses obtained from a single quality characteristic.

Often, in practice, the multiple response problem is formulated as a constrained optimization problem. In general, one response is identified as a *primary response* to be optimized subject to the constraints placed on the remaining responses, which are labeled as *secondary responses*. Design-Expert software package will allow the user to formulate the multiple response optimization problem in this manner. Various mathematical optimization algorithms can be used to solve this problem. The generalized reduced gradient (GRG) method (see [8]) is a very popular choice because the method is broadly applicable to many types of problems with nonlinear equality constraints and because it is commercially available. The popular spreadsheet EXCEL employs a version of the GRG algorithm.

Another way of dealing with multiple responses is to form some function combining the responses, such as $\hat{y} = \sum_{i=1}^{m} w_i \hat{y}_i$, where the w_i are weights, and then optimize the *composite* response $\hat{y}$. In general implementations, the weights are chosen based on the relative importance of the various responses, usually obtained through the advice of experts on the system in question. In practice, choosing the weights appropriately is usually difficult, and so this approach is not widely used unless there is some unequivocal way to select the weights or unless the weights can be selected dynamically via some interactive multiple response optimization procedure. The use of weights to form a composite objective function reduces a multiple problem to a simpler, single problem. However, it also obviously "loses" some information in the conversion.

Recently several authors realized that the constrained function of any performance property could be treated as a new property in its own right as far as multi-objective optimization was concerned. In multi-objective optimization with conflicting objectives, there is no single optimal solution. The interaction among different objectives gives rise to a string of solutions, called Pareto optimal solutions. The decision maker can then have the flexibility of studying the trade-offs involved in selecting the optimal design point. It is also insightful to examine graphically how the controllable variables simultaneously impact the responses [6,7,12].

All of multiple response optimization techniques discussed so far assume that the responses are independent or uncorrelated. There are times in multi-response experiments where the variance–covariance structure of the multiple responses should be taken into account. Khuri and Conlon [4] introduced an approach where a generalized distance measure is used to indicate the weighted distance of each response from its individual optimum value. The weights are determined by the variances and covariances of the responses. Then, the solution is found that minimizes the generalized distance. Pignatiello [13] and Vining [18] proposed a measure based on a squared error loss function which allows the analyst to consider both the specific process economics and the correlation structure. Vining established that the Khuri and Conlon approach is a special case of a weighted squared error loss function, and showed other possible weighting schemes. Even though all the procedures discussed here are sound, they do not provide an extension to robust parameter design where the data are collected from a combined array. Recently, Chiao and Hamada [2] proposed a simple method for improving quality and for achieving robust quality using statistically designed experiments with multiple responses. They defined a *specification region* **S** for the multivariate response as the m dimensional hypercube whose sides are the m component specifications. They considered the covariance matrix of the responses as dependent on the experimental factors and modeled the parameters of the response distribution in terms of these factors. Given a specification region, a measure of quality is the probability that the m component responses are simultaneously meeting their respective specifications or the proportion of conformance. A recent article by Romano et al. [16] adopted a loss function criterion, which is an extension of the mean square error criterion proposed by Vining [18] for the multi-response problem. The authors describe a general framework for multi-response robust design in a combined array structure.