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Closed-form summation of two families of finite tangent sums

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Abstract

In our recent paper with Srivastava [D. Cvijović, H.M. Srivastava, Summation of a family of finite secant sums, Appl. Math. Comput. 190 (2007) 590–598] a remarkably general family of the finite secant sums was summed in closed form by choosing a particularly convenient integration contour and making use of the calculus of residues. In this sequel, we show that this procedure can be extended and we find the summation formulae in terms of the higher order Bernoulli polynomials and the ordinary Bernoulli and Euler polynomials for two general families of the finite tangent sums.
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1. Introduction

Recently, in order to generalize the finite summation problem considered by Chu [1], we have applied the calculus of the residues and summed in closed form a remarkably general family of the finite secant sums [2]. In this sequel, we show that the following two finite tangent sums

$$C_{2n}(q,r) := \sum_{\substack{p=1 \\ p\neq q/2,\ q\ \text{is even}}}^{q-1} \cos\left(\frac{2rp\pi}{q}\right)\tan^{2n}\left(\frac{p\pi}{q}\right) \quad (n\in\mathbb{N}; q\in\mathbb{N}\setminus\{1\}; r=1,\ldots,q-1), \tag{1.1}$$

and

$$S_{2n-1}(q,r) := \sum_{\substack{p=1 \\ p\neq q/2,\ q\ \text{is even}}}^{q-1} \sin\left(\frac{2rp\pi}{q}\right)\tan^{2n-1}\left(\frac{p\pi}{q}\right) \quad (n\in\mathbb{N}; q\in\mathbb{N}\setminus\{1\}; r=1,\ldots,q-1) \tag{1.2}$$

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can be considered in the same way and we find their summation formulae in terms of the higher order Bernoulli polynomials and the ordinary Bernoulli and Euler polynomials.

2. Preliminaries and statement of the results

In what follows, we denote by $B_n^{(m)}(x)$ and $E_n^{(m)}(x)$, respectively, the Bernoulli polynomial of order m and degree n and the Euler polynomial of order m and degree n, defined by means of the following generating functions (see, for details, [4, p. 53 *et seq.*] and [5, Section 1.6])

$$\left(\frac{t}{e^t-1}\right)^m e^{tx} = \sum_{n=0}^{\infty} B_n^{(m)}(x)\frac{t^n}{n!} \quad (|t|<2\pi; m\in\mathbb{N}_0 := \mathbb{N}\cup\{0\}) \tag{2.1}$$

and

$$\left(\frac{2}{e^t+1}\right)^m e^{tx} = \sum_{n=0}^{\infty} E_n^{(m)}(x)\frac{t^n}{n!} \quad (|t|<\pi; m\in\mathbb{N}_0). \tag{2.2}$$

Note that by

$$B_n^{(m)} := B_n^{(m)}(0) \quad (m,n\in\mathbb{N}_0), \tag{2.3}$$

is defined the nth Bernoulli number of the order m. For $m=1$ we have

$$B_n(x) := B_n^{(1)}(x) \quad \text{and} \quad E_n(x) := E_n^{(1)}(x) \quad (n\in\mathbb{N}_0), \tag{2.4}$$

where $B_n(x)$ and $E_n(x)$ are, respectively, the relatively more familiar (ordinary) Bernoulli and Euler polynomials (see, for instance, [4]). The (ordinary) Bernoulli numbers B_n and Euler numbers E_n are given by

$$B_n := B_n(0) \quad \text{and} \quad E_n := 2^n E_n\left(\frac{1}{2}\right) \quad (n\in\mathbb{N}_0). \tag{2.5}$$

We use the floor function $\lfloor x \rfloor$, also called the greatest integer function or integer value, which gives the largest integer less than or equal to x.

Our results are as follows.

Theorem 1. *Let $B_n^{(m)}(x)$ be the Bernoulli polynomial of order m and degree n defined by (2.1) and let $B_n(x)$ and $E_n(x)$ be the (ordinary) Bernoulli and the (ordinary) Euler polynomial defined as in (2.4). Then, the sums $C_{2n}(q,r)$ in (1.1) are given by:*

$$C_{2n}(q,r) = \mathscr{C}_{2n}(q,r) := \begin{cases} \mathscr{C}^o_{2n}(q,r) & \text{if } q \text{ is odd}, \\ \mathscr{C}^e_{2n}(q,r) & \text{if } q \text{ is even}, \end{cases} \tag{2.6}$$

where

$$\mathscr{C}^o_{2n}(q,r) = \frac{(-1)^{n+r}}{2(2n-2)!}\sum_{\alpha=0}^{n-1}\sum_{\beta=0}^{2n}\binom{2(n-1)}{2\alpha}\binom{2n}{\beta}E_{2\alpha+1}\left(\frac{r}{q}\right)B^{(2n)}_{2(n-1)-2\alpha}(\beta)\frac{q^{2\alpha+2}}{2\alpha+1},$$

$$\mathscr{C}^e_{2n}(q,r) = \frac{(-1)^{n+r-1}}{(2n)!}\sum_{\alpha=0}^{n}\sum_{\beta=0}^{2n}\binom{2n}{2\alpha}\binom{2n}{\beta}B_{2\alpha}\left(\frac{r}{q}\right)B^{(2n)}_{2n-2\alpha}(\beta)q^{2\alpha} \quad (n\in\mathbb{N}; q\in\mathbb{N}\setminus\{1\}; r=1,\ldots,q-1).$$

Theorem 2. *Let $B_n^{(m)}(x)$ be the Bernoulli polynomial of order m and degree n defined by (2.1) and let $B_n(x)$ and $E_n(x)$ be the (ordinary) Bernoulli and the (ordinary) Euler polynomial defined as in (2.4). Then, the sums $S_{2n-1}(q,r)$ in (1.2) are given by:*

$$S_{2n-1}(q,r) = \mathscr{S}_{2n-1}(q,r) := \begin{cases} \mathscr{S}^o_{2n-1}(q,r) & \text{if } q \text{ is odd}, \\ \mathscr{S}^e_{2n-1}(q,r) & \text{if } q \text{ is even}, \end{cases} \tag{2.7}$$

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