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A new stage structured predator–prey Gomportz model with time delay and impulsive perturbations on the prey

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Abstract

In this paper, we formulate a robust prey-dependent consumption predator–prey Gomportz model with periodic harvesting (catching or poisoning) for the prey and stage structure for the predator with constant maturation time delay (through-stage time delay) and perform a systematic mathematical and ecological study. By use of the discrete dynamical system determined by the stroboscopic map, we obtain a 'predator-extinction' periodic solution. Further, we show it is globally attractive when some parameters of the system are under appropriate conditions. By using the theory on delay functional and impulsive differential equation, we obtain sufficient condition with time delay for the permanence of the system. In this paper, the main feature is that we introduce time delay and pulse into the predator–prey (natural enemy–pest) model with stage structure, exhibit a new modeling method which is applied to investigate delay impulsive differential equations, and give some reasonable suggestions for pest management.

Keywords: Permanence; Impulsive effect; Predator–prey model with age structure; Maturation time delay; Extinction

1. Introduction

Effectively controlling pests has become an increasingly complex issue over the past two decades. A wide range of pest control ways are available to farmers such as biological, cultural, physical and chemical tools. Farmers often use relatively simple techniques to control the increase in insect pest numbers. For example, farmers often catch the pests by mechanical tools or poison the pests by the overuse of pesticides. However, eradication for the pests is difficult both practically and economically as the pests can breed quickly. Therefore, it may be the best way of ensuring that pest population do not fluctuate widely from one year to the next. All of pests almost have their natural enemies, and these natural enemies can effectively suppress pests sometimes. When people catch or poison the pests, the pests and their natural enemies, which is earlier extinct? For example, field mouses and owls lived in the northwest plain of China in 1998. The owls (natural enemy) became extinct earlier than field mouses when the prey (pest) is catched or poisoned largely, subsequently,

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the survivor (field mouse) increased rapidly and overran ultimately, why? Therefore, we need to investigate the effect of the harvesting for the pests on its natural enemies. In practice, from the principle of ecosystem balance, we need only to control the pest population under the economic threshold level (ETL) and not to eradicate natural enemy totally, and hope the pest population and its natural enemy population can coexist when the pests do not bring about immense economic losses. In this paper, according to the above ecological background, we consider the prey-dependent consumption predator–prey (natural enemy–pest) models with age structure for the predator, the prey is catched or poisoned impulsively, details can be seen in Sections 2 and 5. In Section 3, We prove that, when the impulsive period is no longer than some threshold, i.e. $T < \frac{1}{r}\ln\left[1 - \frac{\ln(1-\delta)}{\ln(\lambda\beta K e^{-d\tau}/d) - \alpha K}\right]$ or partial destruction to pests (preys) by catching or pesticides is at a appropriate extent, i.e. $\delta > 1 - 1/\left(\frac{\lambda\beta K e^{-d\tau}}{d} - \alpha K\right)^{e^{rT}-1}$, the predator-extinction periodic solution is globally attractive, or say, the predator population can be eradicated totally when the pest population is catched or poisoned at a certain extent. However, from the point of ecological balance and saving resources, we only need to control the pest population under the economic threshold level (ETL) in order not to eradicate the predator (natural enemy) totally, so in Section 4, we further prove that, when the impulsive period is longer than the threshold, i.e., $T > -\frac{1}{r}\ln\left\{1 - 1/\left[\ln(1-\delta)\left(\frac{\lambda\beta K e^{-d\tau}}{d} - \alpha K\right)\right]\right\}$ or partial destruction to pests (preys) by catching or pesticides is smaller than the threshold, i.e., $\delta < 1 - 1/\left(\frac{\lambda\beta K e^{-d\tau}}{d} - \alpha K\right)^{1-e^{-rT}}$, pest population and natural enemy population can coexist and the system is permanent.

The predator–prey models with age structure for the predator were introduced or investigated by Hastings, Jiao et al., Wang and Chen, and Gourley and Kuang [1–4]. Since the immature predator takes τ (which is called maturation time delay) units of time to mature, the death toll during the juvenile period should be considered, so, time delays have important biological meanings in age-structured models. Hence many age-structured models with time delay were extensively studied by Wang and Chen, Gourley and Kuang, Liu and Beretta, Liu et al., Song and Cui, Ou et al., Wei and Wang, Aiello and Freedman [3–10]. In recent years, impulsive systems are found in many domains of applied sciences [11–15]. The investigation of impulsive delay differential equations is beginning, and impulsive delay differential equations are almost analyzed in theory by Yan, Leonid and Elena, Liu and Ballinger [16–18]. Time delay and impulse are introduced into predator–prey models with stage structure, which greatly enriches biologic background, but the system become nonautonomous, which causes us greatly difficult in studying the model, thus the literature on global qualitative analysis for delay stage-structured models with impulse effect has never been seen by now. In present paper, we propose a new delay predator–prey Gomportz model with age structure and impulsive effect, study their dynamic behaviors ('predator-extinction' periodic solution, global attractive behavior, permanence).

2. Model and preliminaries

The basic model that we consider is the one based on the idea that predator may consume an increasingly smaller proportion of killed prey as prey density increases. To investigate the effect of this assumption on model, we use the extreme form of prey dependence in predator consumption rate that leads to the hyperbolic Holling II numerical response. From this basic standpoint, to investigate the effect of the harvesting for pests on its natural enemies, motivated by Hastings, Jiao et al., Wang and Chen, Gourley and Kuang, Liu and Beretta, Liu et al., Song and Cui, and Ou et al. [1–8], we give a age-structured predator–prey model with the maturation time delays and impulsive harvesting effect as follows:

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= rx(t)\ln\frac{K}{x(t)} - \frac{\beta x(t)y_2(t)}{1+\alpha x(t)}, \\ \dot{y}_1(t) &= \frac{\lambda\beta x(t)y_2(t)}{1+\alpha x(t)} - e^{-d\tau}\frac{\lambda\beta x(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)} - dy_1(t), \\ \dot{y}_2(t) &= e^{-d\tau}\frac{\lambda\beta x(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)} - dy_2(t), \end{aligned}\right\} \quad t \neq nT, \quad n \in N, \\ \left.\begin{aligned} x(t^+) &= (1-\delta)x(t), \\ y_1(t^+) &= y_1(t), \\ y_2(t^+) &= y_2(t), \end{aligned}\right\} \quad t = nT, \quad n \in N, \end{cases} \tag{1}$$