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The superlinear convergence of a new quasi-Newton-SQP method for constrained optimization

Zengxin Wei [a], Liying Liu [b,*], Shengwei Yao [a]

[a] College of Mathematics and Information Science, Guangxi University, 530004, PR China
[b] Department of Mathematics Science, Liaocheng University, 252059, PR China

Abstract

In this paper, a new variation of the quasi-Newton-SQP methods for constrained optimization is proposed. Under some suitable assumptions, the superlinear convergence of the new method has been established.

Keywords: Quasi-Newton-SQP method; SQP method; Constrained optimization; Superlinear convergence

1. Introduction

In this paper, we consider the constrained optimization problem:

$$\begin{aligned} \min \quad & f(x), \\ \text{s.t.} \quad & h_i(x) = 0, \quad i \in E, \\ & g_j(x) \leqslant 0, \quad j \in I. \end{aligned} \tag{1.1}$$

where $f, h_i, g_j : \Re^n \to \Re$, are twice continuously differentiable functions, $E = \{1, 2, \ldots, m\}$, $I = \{m+1, m+2, \ldots, m+l\}$, Let the Lagrangian function be defined as

$$L(x, \mu, \lambda) = f(x) + \mu^{\mathrm{T}} g(x) + \lambda^{\mathrm{T}} h(x),$$

where μ and λ are multipliers. Obviously, the Lagrangian function L is a twice continuously differentiable function. Let S be the feasible point set of the problem (1.1). We defined $I^\star$ to be the set of all the subscripts of those inequality constraints which are active at $x^\star$, i.e.,

$$I^\star = \{i \mid i \in I \text{ and } g_i(x) = 0\}.$$

SQP methods for solving twice continuously differentiable nonlinear programming problems, are essentially Newton-type methods for finding Kuhn–Tucker points of nonlinear programming problems. Recently, this

* Corresponding author.
E-mail addresses: lyliu001@163.com (L. Liu), idhot@sina.com (S. Yao).

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methods have been in vogue [1–3,7–10]: Powell [1] gave the BFGS-Newton-SQP method for the nonlinearly constrained optimizations. He gave some sufficient conditions, under which SQP method would yield 2-step Q-superlinear convergence rate (assuming convergence) but did not show that his modified BFGS method satisfied these conditions. Instead R-superlinear convergence was proofed. Coleman and Conn [2] gave a new local convergence quasi-Newton-SQP method for the equality constrained nonlinear programming problems. The local 2-step Q-superlinear convergence was established. W. Sun gave quasi-Newton-SQP method for general LC^1 constrained optimization problems [3]. He gave the locally convergent sufficient conditions and superlinearly convergent sufficient conditions. But he did not prove whether the modified BFGS-quasi-Newton-SQP method satisfies the sufficient conditions or not.

In this paper, we will present a modified quasi-Newton-SQP method for constrained optimization (1.1), and use the modified BFGS update formula [4]

$$B_{k+1}(3) = B_k(3) - \frac{B_k(3)s_k s_k^{\mathrm{T}} B_k(3)}{s_k^{\mathrm{T}} B_k(3) s_k} + \frac{y_k(3) y_k(3)^{\mathrm{T}}}{y_k(3)^{\mathrm{T}} s_k}, \tag{1.2}$$

where $s_k = x_{k+1} - x_k$ and $y_k(3) = g_{k+1} - g_k + A_k(3)s_k$, $A_k(3) = \frac{2[f(x_k)-f(x_{k+1})]+(g_{k+1}+g_k)^{\mathrm{T}}s_k}{\|s_k\|^2} I$. In this paper, $\mathrm{MBFGS}^\bigstar$ update formula is defined as follows:

$$B_{k+1}^\bigstar = B_k^\bigstar - \frac{B_k^\bigstar \tilde{s}_k \tilde{s}_k^{\mathrm{T}} B_k^\bigstar}{\tilde{s}_k^{\mathrm{T}} B_k^\bigstar \tilde{s}_k} + \frac{y_k^\bigstar {y_k^\bigstar}^{\mathrm{T}}}{{y_k^\bigstar}^{\mathrm{T}} \tilde{s}_k}, \tag{1.3}$$

where $\tilde{s}_k = z_{k+1} - z_k = (x_{k+1}, \mu_{k+1}, \lambda_{k+1}) - (x_k, \mu_k, \lambda_k)$, $\tilde{y}_k = \nabla_x L(z_{k+1}) - \nabla_x L(z_k)$ and $y_k^\bigstar = \tilde{y}_k + A_k^\bigstar \tilde{s}_k$, $A_k^\bigstar = \frac{2[L(z_k)-L(z_{k+1})]+[\nabla_x L(z_{k+1})+\nabla_x L(z_k)]^{\mathrm{T}}\tilde{s}_k}{\|\tilde{s}_k\|^2} I$, in this place, μ_k and λ_k are the multipliers which are according to the objective function at x_k, while μ_{k+1} and λ_{k+1} are the multipliers which are according to the objective function at x_{k+1}. Therefore, we can get a new $\mathrm{MBFGS}^\bigstar$–SQP algorithm for constrained optimization. Under some sufficient conditions we will establish the superlinear convergence of the given method.

This paper is organized as follows. In the next section, we will give the new $\mathrm{MBFGS}^\bigstar$–SQP algorithm for constrained optimization problems. In Section 3, the superlinear convergence of the new method will be established.

2. New algorithm

The first-order Kuhn–Tucker condition of problem (1.1) is

$$\begin{cases} \nabla f(x^\bigstar) + \mu^{\mathrm{T}} \nabla g(x^\bigstar) + \lambda^{\mathrm{T}} \nabla h(x^\bigstar) = 0, \\ g(x^\bigstar) \leqslant 0, \quad \mu_j \geqslant 0, \quad \mu_j g_j(x^\bigstar) = 0, \quad \text{for } j \in I, \\ h(x^\bigstar) = 0. \end{cases} \tag{2.1}$$

The above system (2.1) can be represented by the following system:

$$H(z) = 0, \tag{2.2}$$

where $z = (x, \mu, \lambda) \in S$ and $H : \Re^{n+m+l} \to \Re^{n+m+l}$ is defined by

$$H(z) = \begin{pmatrix} \nabla f(x) + \mu^{\mathrm{T}} \nabla g(x) + \lambda^{\mathrm{T}} \nabla h(x) \\ \min\{\mu, -g(x)\} \\ h(x) \end{pmatrix}. \tag{2.3}$$

Since $\nabla f, \nabla g$ and ∇h are continuously differentiable functions, it is obviously that $H(z)$ is continuously differentiable function. Then for all $\tilde{d} \in \Re^{n+m+l}$, the directional derivative $H'(z : \tilde{d})$ of the function $H(z)$ exists.

Now we define the index sets

$$\alpha(z) = \{i | \mu_i > -g_i(x)\} \tag{2.4}$$

and

$$\beta(z) = \{i | \mu_i \leqslant -g_i(x)\}. \tag{2.5}$$