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Raman threshold for *n*th-order cascade Raman amplification

L. de la Cruz-May [a], J.A. Álvarez-Chavez [b,*], E.B. Mejía [c], A. Flores-Gil [a], F. Mendez-Martinez [a,b,c,d], S. Wabnitz [d]

[a] Universidad Autónoma del Carmen, Facultad de Ingeniería, C. 56, No. 4. 24180, Cd. del Carmen, Campeche, Mexico
[b] Centro de Investigación e Innovación Tecnológica, IPN, Cerrada CECATI S/N, Sta. Catarina, Del. Azcapotzalco, 02250 México D.F., Mexico
[c] Centro de Investigaciones en Óptica, Loma del bosque 115, 37150 León, Gto., Mexico
[d] Dipartamento di Ingegneria dell'Informazione, Università di Brescia, via Branze 38, Brescia, Italy

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ABSTRACT

We study theoretically and experimentally Raman threshold for 1, 2, ... , *n* orders Stokes in a free running configuration. Using alternative way to solve the differential coupled equations that describe the stimulate Raman scattering, we find simple mathematical expressions that allow calculating the necessary pumping power to obtain Raman threshold for *n*th-order Stokes and the maximum output power available in each Stokes. The theoretical calculations coincide with the results obtained experimentally.

1. Introduction

Cascaded Raman amplification in the wavelength region 1.1–1.7 μm has been shown as a promising technique to improve the performance in long span optical communications systems. In order to obtain *n*th-order Raman amplification, several pump schemes have been reported. The two main choices range from a scheme on which, using commercial communications fibers as the Raman gain medium are spliced to fiber Bragg gratings (FBG) [1,2] to form the resonator, and as free running which use Si-doped or Ge-doped fibers as Raman gain medium [3,4]. In this paper we report a theoretical and experimental study of cascaded Raman amplification in silica fiber in free running configuration. The term cascade Raman refers to the fact that an initial intense beam coupled into the fiber is sequentially-converted in Stokes components via stimulated Raman scattering (SRS) as it propagates. The mechanism SRS arises when a small fraction (typically $\sim 10^{-6}$) of an intense laser beam that propagates along an optical fiber is Raman-scattered i.e. pump photons instantaneously excite the energy of the glass lattice up to a virtual level from where it decays instantaneously to a vibration mode that has got higher than the initial vibration state [5]. The energy radiated by this process corresponds to Stokes photons. From this level, the lattice makes a discrete return to its initial vibration state liberating phonons that are resonant with the lattice and thus reabsorbed instead of radiated. This is the spontaneous Raman scattering (RS). The few scattered photons that propagate through the fiber-axis are amplified as long as population inversion exists. The fulfillment of population inversion condition only requires having population in the virtual level because this populates the high-energy vibration mode. This is the stimulated version of the RS. Then, an intensely pumped medium with population on the virtual level has the ability to amplify a Stokes signal. As the Stokes signal grows, the pump decreases until vanishing. Now, this signal may be sufficient to produce the next Stokes by SRS and so on. When this sequence occurs in an optical fiber, it becomes a cascaded Raman amplifier.

Taking the coupled equations that describe the stimulated Raman scattering process [6] we study the Raman threshold condition and the highest power reached for the first Stokes wave. This analysis is then extended for *n*th-order Stokes wave and we propose relationships to determine the *n*th-order cascaded Raman amplification.

2. Analysis

In the case of a continuous wave regime, the pump power propagation and Stokes wave in a single mode fiber in the same direction is described by the following equations [6]:

$$dP_P/dz = -\alpha_P P_P - (\nu_P/\nu_S)(g_R/A_{eff})P_P P_F \quad (1)$$

$$dP_F/dz = -\alpha_S P_F + (g_R/A_{eff})P_P P_F \quad (2)$$

where P_P and P_F are the power of the pumped and Stokes respectively, g_R is the Raman gain coefficient, α_P and α_S are the fiber loss coefficient at pumped and signal wavelength respectively, ν_P and ν_S are the frequency of the pump wave and Stokes signals respectively. Note that within the transparency windows of the losses

* Corresponding author.
E-mail address: jalvarezch@ipn.mx (J.A. Álvarez-Chavez).

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spectrum of silica fiber $\alpha_P \approx \alpha_s \approx \alpha$. Thus, it is possible to obtain a solution of the couple of equations described above by means of the following procedure: first we divide Eq. (1) with Eq. (2), second we have to add again Eq. (1) and Eq. (2), and finally we obtain two separable differential equations given by:

$$[(-\alpha + (g_R P_P/A_{eff}))](dP_P/P_P) = [(-\alpha - (\nu_P g_R P_F/(\nu_S A_{eff})))](dP_F/P_F) \quad (3)$$

$$(d/dz)[(\nu_S/\nu_P)P_P + P_F] = -\alpha((\nu_S/\nu_P)P_P + P_F) \quad (4)$$

We consider that the pumping intensity moves inside the silica fiber from z = 0 to z = L. Solutions to the differential Eqs. (3) and (4) are given respectively by:

$$\ln((P_F P_{P0})/(P_P P_{F0})) = (\nu_P g_R)/(\alpha \nu_S A_{eff})[((\nu_S/\nu_P)P_{P0} + P_{F0}) - (((\nu_S/\nu_P)P_P + P_F)] \quad (5)$$

$$((\nu_S/\nu_P)P_P + P_F) = ((\nu_S/\nu_P)P_{P0} + P_{F0})\exp(-\alpha z) \quad (6)$$

where P_{P0} and P_{F0} are the pumping power and forward Stokes in $z = 0$, P_{PL} and P_{FL} are the pumping power and the forward Stokes power at the end of the fiber in $z = L$. Substituting Eq. (6) in Eq. (5), we obtain

$$\ln((P_F P_{P0})/(P_P P_{F0})) = (\nu_P g_R)/(\alpha \nu_S A_{eff})((\nu_S/\nu_P)P_{P0} + P_{F0})[1 - \exp(-\alpha z)/\alpha] \quad (7)$$

To the best of our knowledge, this approach has not been taken before, for instance Smith in his work has solved the equations for the SRS neglecting the second term of Eq. (1) that this responsible for the pump depletion [7].

Ignoring P_{F0} in the second member of Eq. (7) for the reason that $P_{F0} \ll \nu_s/\nu_p P_{P0}$, a simple equation that governs the interaction between pump power and the Stokes power is obtained:

$$\ln((P_F P_{P0})/(P_P P_{F0})) = \frac{g_R}{A_{eff}} P_{P0} L_{eff} \quad (8)$$

where L_{eff} is the effective fiber length given by $(1 - \exp(-\alpha z))/\alpha$.

Pump power is coupled at the input end, and so the spontaneous Raman scattering acts as a probe and is amplified along with the forward propagation. After some consideration we proposed a transferred power of $\sim 10^{-7}$ to spontaneous Raman scattering, i.e., $P_{F0} \approx 10^{-7} P_{P0}$. The propagation of the pump power and the Stokes signal throughout the fiber is described via Eq. (8). Two most important points for investigation are the Raman threshold and the maximum output power available in first-order Stokes peak. These special points are shown in Fig. 1, as A and B, respectively.

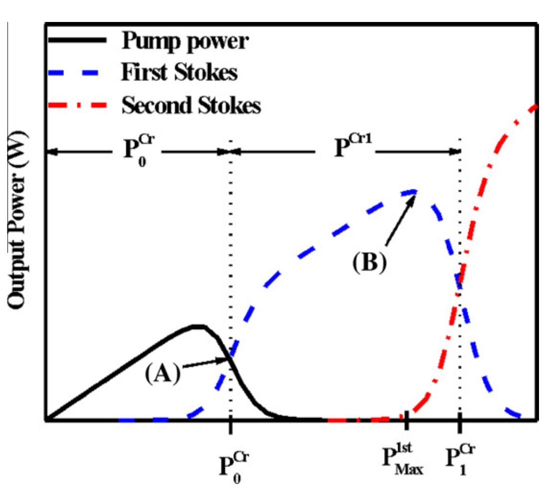

Fig. 1. First and second Raman thresholds.

Fig. 1 thoroughly describes the relation between coupled pump power for Raman threshold and maximum power for first Stokes, including the subsequent Raman threshold.

Raman threshold has been defined as the pump power level at which P_P and P_F are equal at the output end of the fiber. Taking in consideration this condition and substituting in Eq. (8), the P_0^{Cr} critical power is obtained as:

$$P_0^{Cr}(g_R L_{eff}/A_{eff}) \approx l6 \quad (9)$$

This relationship was already previously employed by Smith [7]. Substituting Eq. (9) in Eq. (6) we find the output power at which critical power is reached, given for

$$P_{RT} \approx 8A_{eff}/(g_R L_{eff})\exp(-\alpha L) \quad (10)$$

Once the Raman threshold is reached, first Stokes grows quickly until the pump power is unable to continue transferring power. We propose that the transfer concludes when the residual pump power is approximately 10^{-6} of the first Stokes power, i.e., $P_{PL} \approx 10^{-6} P_{FL}$. With this assumption we estimate the pumping power for which first Stokes reaches the higher available power before transferring power to the second Stokes as:

$$P_{Max}^{1st}(g_R L_{eff}/A_{eff}) \approx 30 \quad (11)$$

Substituting Eq. (11) in Eq. (6) we obtained the maximum power reached by first Stokes peak:

$$P_1^{Max} \approx 30(A_{eff}/(g_R L_{eff}))\exp(-\alpha L) \quad (12)$$

When first-order Stokes reaches its maximum power, it will eventually produce the RS necessary to generate the second-order Stokes, see Fig. 1. We now proceed to calculate the Raman threshold for second Stokes by re-writing the coupled Eqs. (1) and (2) as follows

$$dP_F/dz = -\alpha_S P_F - (\nu_s/\nu_{s2})(g_{R2}/A_{eff2})P_F P_{F2} \quad (13)$$

$$dP_{F2}/dz = -\alpha_{S2} P_{F2} - (\nu_S/\nu_{S2})(g_{R2}/A_{eff2})P_F P_{F2} \quad (14)$$

Following the same procedure used to derive Eq. (8), we find the Raman threshold for second Stokes to be:

$$P^{Cr1}(g_{R2}L_{eff2}/A_{eff2}) \approx 16 \quad (15)$$

where A_{eff2} represents the effective area for first Stokes wave, g_{R2} is the Raman gain coefficient for second Stokes wave, L_{eff2} is the effective fiber length represented for $(1 - \exp(-\alpha_s L_{eff}))/\alpha_s$ and P^{Cr1} is the necessary first Stokes power to reach the Raman threshold of second Stokes. Therefore, the pumping power necessary to reach the Raman threshold of the second Stokes is given by the sum Eqs. (9) and (15), see Fig. 1. Therefore,

$$\begin{aligned} P_2^{Cr} &= P_0^{Cr} + P^{Cr1} \\ &= 16(A_{eff}/g_R L_{eff}) + 16(A_{eff2}/g_{R2}L_{eff2}) \\ &\approx 32A_{eff}/(g_R L_{eff}) \end{aligned} \quad (16)$$

In Eq. (16), we suppose that $A_{eff}/(g_R L_{eff}) \approx A_{eff2}/(g_{R2}L_{eff2})$.

In general we provide two mathematical expressions to calculate the critical power and the maximum power of the nth-order Raman amplification in a silica fiber given by

$$P_N^{Cr} \approx 16 * N * A_{eff}/(g_R L_{eff}) \quad (17)$$

$$P_N^{Max} \approx 30 * N * A_{eff2}/(g_{R2}L_{eff2})\exp(-\alpha L) \quad (18)$$

where P_N^{Cr} and P_N^{Max} represents the necessary pumping power to reach the Raman threshold and the maximum output power of the nth-order Stokes, respectively. With the support of Eqs. (17) and (18) we are able to calculate the Raman threshold and maximum power of the nth-order Stokes.