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The Euler scheme for random impulsive differential equations ☆

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Abstract

Random impulsive differential equations (RIDEs) are a kind of mathematical models with extensive applications. In this paper, the Euler scheme for RIDEs is first brought forward, one of whose important applications is to generate the whole approximate trajectories of RIDEs. Thus the proposed Euler scheme allows us to approximate moments, functionals and the distribution for the underlying process and perform Monte-Carlo type analysis. The obtained results show that the Euler scheme is at least 1-order of step h when the right terms of equation satisfy Lipschitz conditions and the waiting times of random impulses follow the mutually independent exponential distribution with the same parameter λ. Thus it is an efficient method for numerical simulation.

Keywords: Differential equation; Random impulse; Euler scheme; Continuous dependence on initial value; Convergence

1. Introduction

Impulsive differential equations are adequate mathematical models for numerous processes and phenomena studied in population dynamics [2], physics and chemistry [3], and engineering [4], etc. Significant progress has been made in the theory of impulsive differential equations in recent years. In effect, the theory of impulsive differential equations is considerably richer than the theory of ordinary differential equations, see [8] and references therein.

Up to now, most known literatures investigate impulsive differential equations with two kinds of impulse times: fixed impulse times and varying impulse times, where so-called varying impulse time means that the time that impulse happens is some functions of "state x", see [1,6,9,10,13]. The two kinds of impulse times are deterministic. However, actual impulse does not always happen at deterministic time but usually at random time, that is, impulse time t_k is a random variable, $k = 1, 2, \ldots$ For example, consider an interest rate model. The time when interest rate is adjusted is a random variable. However, interest rate is a constant during two

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neighboring adjusted times. Thus, the interest rate $r(t)$ can be modeled by random impulsive differential equation

$$\begin{cases} \frac{dr(t)}{dt} = 0, & t \neq t_k, \\ r(t_k) = I_k(r(t_k^-)), & k = 1, 2, \ldots, \end{cases}$$

where $\{t_k\}$ denote the times that the interest rate is adjusted, which are a series of random variables. I_k is some pending function of $r(t_k)$. Thus, random impulsive differential equations have their value of applications. In fact, Iwankievicz and Nielsen [5] investigated dynamic response of non-linear systems to Poisson distributed random impulses. Tatsuyuki et al. [14] presented a random impulse model to depict drift motion of granules in chara cells due to myosin–actin interaction, which is a descriptive model but a mathematical model. Sanz-Serna and Stuart [12] first brought forward dissipative differential equations with random impulses and used Markov chains to simulate such systems. Wu and Meng [16] first gave general *random impulsive differential equations* (RIDEs)

$$\begin{cases} x'(t) = f(t, x(t)) \text{ a.e.}, & t \neq \tau_k, \\ \Delta x(\tau_k^+) = I_k(\tau_k, x(\tau_k)) \text{ a.e.}, & k = 1, 2, \ldots, \end{cases}$$

where τ_k is the kth impulse moment, which is a random variable, $k = 1, 2, \ldots$ From then on, several important properties of RIDEs have been investigated. Wu and Meng [16] discussed p-moment boundedness of the above equation by the second Liapunov method. Wu and Duan [17] discussed oscillation, stability, and boundedness in mean square of second-order linear RIDEs using comparing them with those of the corresponding differential equation without impulsive effect. Wu and Han [18] investigated p-moment exponential stability of RIDEs by the second Liapunov's method. Wu et al. [19] studied the existence and uniqueness of solutions to RIDEs.

RIDEs have no simple explicit solutions or known distributions. In practice, some functionals that is the mathematical expectations of functions of solutions of RIDEs, have to be computed. It is well-known that Monte-Carlo simulation methods and Markov chain methods have been developed as a powerful methodology to overcome the evaluation problem for stochastic differential equations, see [7,11,12,15] and their references. The efficiency of Monte-Carlo methods strongly depends on the use of appropriate discrete time approximations. In this paper, we first bring forward the Euler scheme for RIDEs, one of whose important applications is to generate the whole approximate trajectories of RIDEs. Thus the proposed Euler scheme allows us to approximate moments, functionals and the distribution for the underlying process and perform Monte-Carlo type analysis.

The structure of this paper follows as: The Euler scheme will be presented in Section 2. Section 3 completes algorithmic analysis. An example and conclusions are given in Sections 4 and 5, respectively.

2. The Euler scheme

In this section, we will bring forward the Euler scheme for random impulsive differential equations. For the sake of simplicity, we first denote

$$\mathbf{R} = (-\infty, +\infty), \quad \mathbf{R}_+ = [0, +\infty), \quad \mathbf{R}_\tau = [\tau, +\infty),$$

where $\tau \in \mathbf{R}$ is a constant. Let w_k be a random variable defined in $D_k \equiv (0, d_k)$, where $d_k \in (0, +\infty]$, $k = 1, 2, \ldots$ Furthermore, we assume that w_i and w_j are mutually independent as $i \neq j$ for all $i, j = 1, 2, \ldots$

Consider random impulsive differential equations

$$\begin{cases} x'(t) = f(t, x(t)) \text{ a.e.}, & t > t_0, \ t \neq \tau_k, \\ x(\tau_k) = I_k(\tau_k^-, x(\tau_k^-)) \text{ a.e.}, & k = 1, 2, \ldots, \end{cases} \tag{1}$$

where $f : \mathbf{R}_\tau \times \mathbf{R}^n \to \mathbf{R}^n$. $\tau_1 = t_0 + w_1$ and $\tau_k = \tau_{k-1} + w_k$ as $k = 2, 3, \ldots$, herein $t_0 \in \mathbf{R}_\tau$ and w_k denotes the waiting time that the solution to system (1) jumps after the $(k-1)$th jump. $x(\tau_k^-) \equiv \lim_{t \to \tau_k - 0} x(t)$. $I_k : R_\tau \times \mathbf{R}^n \to \mathbf{R}^n$. "a.e." is the abbreviation of "almost everywhere".