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Design of fixed-lag smoother using covariance information based on innovations approach in linear discrete-time stochastic systems

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Abstract

This paper newly designs the recursive least-squares (RLS) Wiener fixed-lag smoother and filter using the covariance information in linear discrete-time stochastic systems. The estimators require the information of the observation matrix, the system matrix for the state variable, related with the signal, the variance of the state variable, the cross-variance function of the state variable with the observed value and the variance of the white observation noise. It is assumed that the signal is observed with additive white noise. The current fixed-lag smoothing algorithm has a characteristic, as shown in Theorem 1, that the fixed-lag smoothing estimate of the state vector is calculated in the reverse direction of time.

Keywords: Wiener–Hopf equation; Linear stochastic systems; Recursive Wiener filter; Covariance information; Fixed-lag smoother

1. Introduction

The estimation problem given the covariance information has been seen as an important research in the area of the detection and estimation problems for communication systems. In [1–3], it is assumed that the auto-covariance function of the signal is expressed in the semi-degenerate kernel form. The semi-degenerate kernel is the function suitable for expressing a general kind of auto-covariance function by a finite sum of non-random functions. In the fixed-lag smoother [4], the auto-covariance function of the signal is expressed in the degenerate kernel form, not in the semi-degenerate kernel form. The degenerate kernel function cannot express the auto-covariance functions of general kinds of stochastic processes. Hence, the fixed-lag smoother using the auto-covariance function in the form of the degenerate kernel is not appropriate for estimating the general kinds of stationary or non-stationary signal processes. The expression in the degenerate kernel form of

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the auto-covariance function is obtained through approximating the auto-covariance function by the Fourier series expansion. Hence, its approximation error causes the degradation in the estimation accuracy of the fixed-lag smoother. The recursive least-squares (RLS) Wiener fixed-point smoother [5] and filter [6] using the covariance information are designed in linear discrete-time stochastic systems. The estimators require the information of the observation matrix, the system matrix for the state variable, related with the signal, the variance of the state variable and the cross-variance function of the state variable with the observed value. These information can be obtained from the covariance function of the signal [5]. Also, it is assumed that the variance of the white observation noise is known.

From this respect, this paper newly designs the RLS fixed-lag smoothing and filtering algorithms using the covariance information in linear discrete-time stochastic systems. It is assumed that the signal is observed with additional white observation noise. The estimators require the information of the factorized auto-covariance function of the signal. Namely, the observation matrix, the system matrix for the state variable, the variance of the state variable and the cross-variance function of the state variable with the signal are used as in [5]. Also, it is assumed that the variance of the white observation noise is known. Usually, the fixed-lag smoother has better estimation accuracy than the filter, since more observed values of finite number are used in comparison with the filter. Theorem 1 proposes the algorithm, which calculates the fixed-lag smoothing estimate of the signal and the state vector. The current fixed-lag smoothing algorithm has a characteristic, as shown in Theorem 1, that the fixed-lag smoothing estimate of the state vector is calculated in the reverse direction of time. The fixed-lag smoothing and filtering algorithms are derived based on the invariant imbedding method [5]. The fixed-lag smoothing error variance function is also formulated.

A numerical simulation example is demonstrated to show the validity of the proposed fixed-lag smoother. Also, the estimation accuracy of the proposed fixed-lag smoother is compared with the fixed-point smoother and the filter in [5].

2. Fixed-lag smoothing problem

Let an observation equation be given by

$$y(k) = z(k) + v(k), \quad z(k) = Hx(k) \tag{1}$$

in discrete-time stochastic systems, where $z(k)$ is an $m \times 1$ signal vector, $x(k)$ is an $n \times 1$ state variable, H is an $m \times n$ observation matrix and $v(k)$ is white observation noise. It is assumed that the signal and the observation noise are mutually independent and that $z(k)$ and $v(k)$ are zero mean. Let the auto-covariance function of $v(k)$ be given by

$$E[v(k)v^{\mathrm{T}}(s)] = R(k)\delta_K(k-s), \quad R(k) > 0. \tag{2}$$

Here $\delta_K(\cdot)$ denotes the Kronecker δ function.

The fixed-lag smoothing estimate is expressed by

$$\hat{x}(k,k+L) = \sum_{i=1}^{k} g(k,i)\upsilon(i) + \sum_{i=k+1}^{k+L} g(k,i)\upsilon(i) \tag{3}$$

as a linear transformation of the innovations process $\{\upsilon(i), 1 \leqslant i \leqslant k+L\}$, where $g(k,s)$ and L are referred to be an impulse response function and the fixed lag. The innovations process is expressed by $\upsilon(k) = y(k) - H\Phi\hat{x}(k-1,k-1)$, where $\hat{x}(k-1,k-1)$ is a filtering estimate of $x(k-1)$ and Φ represents the system matrix in the state equation for $x(k)$. The auto-covariance function of the innovations process is given by [7]

$$E[\upsilon(k)\upsilon^{\mathrm{T}}(s)] = \Pi(k)\delta_K(k-s). \tag{4}$$

The first term on the right hand side of (3) represents the filtering estimate

$$\hat{x}(k,k) = \sum_{i=1}^{k} g(k,i)\upsilon(i) \tag{5}$$

of $x(k)$ and the second term is the correction quantity for the fixed-lag smoothing estimate [7].